SLR-HO-1

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Group and Ring Theory (2317101)

Day & Date: Friday, 10-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) Consider the following statements
P: Two subnormal series of a group G have isomorphic refinements.
Q: Any two composition series of a group G are isomorphic.
   a) P is true and Q is false
   b) P is false and Q is true
   c) Both P and Q are true
   d) Both P and Q are false

2) Which of the following is a ring?
   a) $(\mathrm{Z}, +, .)$
   b) $(\mathrm{Q}, +, .)$
   c) $(\mathrm{R}, +, .)$
   d) All of these

3) Two elements $a, b \in G$ are conjugate if for any $g \in G$ such that ______.
   a) $b = gag$
   b) $b = g^{-1}ag$
   c) $a = gag^{-1}$
   d) $b = g^{-1}a\, g^{-1}$

4) Which of the following is not a field?
   a) Z
   b) Q
   c) R
   d) C

5) In a ring of integers associate of 5 are ______.
   a) 5 and -5
   b) 5 and 0
   c) 1 and -1
   d) 0

6) A commutative ring which has no zero divisors is called ______.
   a) Field
   b) Division ring
   c) Integral domain
   d) None of these

7) Number of subgroups of $Z_6 =$ ______.
   a) 3
   b) 6
   c) 4
   d) None of these

8) Pick the incorrect statement?
   a) Every group possess at least two normal subgroups
   b) Intersection of two normal subgroup of group G is normal
   c) Every subgroup of Abelian group is normal
   d) Q_8 is abelian group

B) True or False. **04**

1) Two Sylow p subgroup of a group G are conjugate to each other.
2) A group having no proper normal subgroup is called simple group.
3) Every group of prime order is non-abelian.
4) The cyclotomic polynomial $\varphi_p = \frac{x^{p-1}}{x-1} = x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over Q for any prime p.

Q.2 Answer the following. (Any Six) **12**

a) State Division algorithm in $F[x]$.

b) Define:
1) Unit
2) Associate

c) Explain concept of irreducible polynomial with one example.

d) Define:
1) Prime ideal
2) Maximal ideal

e) Explain the term: content of polynomial.

f) If G is a group and S be any non-empty subset of G then prove that $N[S]$ is subgroup of G.

g) Prove or disprove $Z[\sqrt{-5}]$ is UFD.

h) If G is a group and G be the derived subgroup of G then prove that G is normal subgroup in G.

Q.3 Answer the following. (Any three) **12**

a) Prove that: Every homomorphic image of nilpotent group is nilpotent.

b) Prove that every principal ideal domain is unique factorization domain.

c) If G be a finite group with $|G| = p^n$ where p is prime number then prove that the center of G is non trivial.

d) If G is a group and G' is the derived subgroup of G then prove that G' is a normal subgroup of G.

Q.4 Answer the following. (Any two) **12**

a) Prove that: A group G is said to be solvable iff there exists some positive integer k such that $G^k = \{e\}$.

b) Prove that: No group of order 30 is simple.

c) If F is a field then prove that the ideal generated by $p(x) \neq 0$ of $f(x)$ is maximal iff $p(x)$ is irreducible over F.

Q.5 Answer the following. (Any two) **12**

a) State and prove Eisenstein's criteria of irreducibility over Q.

b) If R be a ring then prove that $R[x]$ has unity iff R has unity.

c) If X be any G-set then prove that $|xG| = (G: Gx)$
For any $x \in X$ where $(G: Gx)$ is index of Gx in G.

SLR-HO-2

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Real Analysis (2317102)

Day & Date: Monday, 13-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) If f_1 and f_2 are bounded and integrable functions on $[a, b]$ then the following function/functions is/are integrable.
   a) $f_1 - f_2$ b) f_1^2
   c) $|f_1|$ d) All of the above

2) Consider the following statements:
   I) Every monotonic increasing function on $[a, b]$ is bounded.
   II) Every monotonic increasing function on $[a, b]$ is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

3) If f is non-negative function on $[a, b]$ such that $\int_0^1 f(x)dx = 0$ then _____ for all $x \in [a, b]$.
   a) $f(x) = 0$ b) $f(x) \geq 0$
   c) $f(x) \leq 0$ d) $f(x)$ do not exist

4) $\int_a^b kdx =$_____ where k is a constant.
   a) $k(b - a)$ b) $k(b + a)$
   c) $k(a - b)$ d) k

5) The value of M and m for $f(x) = x$ on $[1,2]$ are $M =$_____, $m =$ _____.
   a) 1,2 b) 2,1
   c) 1,1 d) 1,0

6) If f is integrable over $[a, b]$ then _____ $\leq \int_a^b f(x)dx \leq$ _____.
   a) $m(b - a), M(b - a)$ b) $m(b + a), M(b + a)$
   c) $M(b - a), m(b - a)$ d) $M(b + a), m(b + a)$

7) The oscillatory sum of a function f over $[a, b]$ is defined as $W(P, f) =$ _____.
   a) $\sum_{i=1}^{n} M_i \Delta x_i$ b) $\sum_{i=1}^{n} m_i \Delta x_i$
   c) $\sum_{i=1}^{n} (M_i - m_i)\Delta x_i$ d) $\sum_{i=1}^{n} (M_i + m_i)\Delta x_i$

8) Consider the following statements:
   I) Function having only one point discontinuity is integrable.
   II) Function having finite no. of points of discontinuity is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

B) Fill in the blanks. **04**

1) Riemann - Stieltje's integral reduces to Riemann integral if $\alpha(x) =$ _____
2) The upper integral of a function f on $[a, b]$ is defined as _____.
3) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is given by $P^* =$ _____.
4) The Riemann Sum is given by $S(P, f) =$ _____.

Q.2 Answer the following. (Any Six) **12**

a) Define: Upper Integral and Lower Integral.

b) Write short note on Primitive of function.

c) State second Fundamental theorem of Integral Calculus.

d) Find the directional derivative of $(f(x, y) = x^3 + xy$ at point $(1,3)$ in the direction $(1, -1))$.

e) State condition of integrability of Riemann Stieltjes Integral.

f) Define: Directional derivative.

g) Write Mean Value theorem for the functions f from $R^n \to R$.

h) Check whether the function $f(x) = x^2 + 4x + 3$ have local extrema or not.

Q.3 Answer the following. (Any Three) **12**

a) If a function f is continuous on $[a, b]$ then prove that there exists a number ξ in $[a, b]$ such that $\int_a^b f(x)dx = f(\xi)(b - a)$.

b) Solve $\int_1^5 (3x + 5)dx$ by Riemann sum method.

c) Prove that: The oscillation of a bounded function f on an interval $[a, b]$ is the supremum of the set $\{|f(x_1) - f(x_2)|/x_1, x_2 \in [a, b]\}$ of numbers.

d) Check whether directional derivative of following function exists at 0 in the direction of $u = (u_1, u_2)$

$$f(x) = \begin{cases} \dfrac{x.y}{x+y} & \text{if}(x, y) \neq (0,0) \\ 0, & \text{if}(x, y) = (0,0) \end{cases}$$

Q.4 Answer the following. (Any Two) **12**

a) If f and all its partial derivatives of order less than m are differentiable at each point of an open set S in R^n and a, b are two points of S such that $L(a, b) \subseteq S$ then prove that there is a point z on the line segment $L(a, b)$ such that $f(b) - f(a) = \sum_{k=1}^{m-1} \frac{1}{k!} f^{(k)}(a; b - a) + \frac{1}{m!} f^m(z; b - a)$

b) If f have a continuous n^{th} (for some integer $n \geq 1$) derivative in the open interval (a, b) and for some interior point c in (a, b) we have, $f'(c) = f''(c) = \cdots = f^{n-1}(c) = 0$ but $f^n(c) \neq 0$ then prove that for n even, f has local minimum at c if $f^n(c) > 0$ and f has local maximum at c if $f^n(c) < 0$. Also prove that if n is add, there is neither a local maximum nor a local minimum at c.

c) If f is integrable on $[a, b]$ then prove that f^2 is also integrable on $[a, b]$.

Q.5 Answer the following. (Any Two) **12**

a) If f is bounded function on $[a, b]$ then prove that for every $\epsilon > 0$ there corresponds $\delta > 0$ such that

1) $$U(P, f) < \int_a^{\bar{b}} f(x)dx + \epsilon$$

2) $$L(P, f) > \int_{\underline{a}}^{b} f(x)dx - \epsilon$$

for every partition P of $[a, b]$ with norm $\mu(P) < \delta$.

b) If a function f is monotonic on $[a, b]$ then prove that f is integrable.

c) If a function f is bounded and integrable on $[a, b]$ then prove that the function F defined as, $F(x) = \int_a^x f(t)dt; a \leq x \leq b$ is continuous on $[a, b]$. Furthermore if f is continuous at a point c of $[a, b]$ then prove that F is derivable at c and $F'(c) = f(c)$.

SLR-HO-3

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Number Theory (2317107)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) The congruence $x^2 \equiv -1(mod\ p)$, p is a prime, has a solution iff _____.
   a) $p \equiv -1(mod\ 4)$ b) $p \equiv 0(mod\ 4)$
   c) $p \equiv 1(mod\ 4)$ d) $p \equiv 1(mod\ p^2)$

2) If a and b are integers, p is a prime such that $\mathrm{p}|\mathrm{ab}$ and $\mathrm{p} \nmid \mathrm{b}$ then _____.
   a) $\mathrm{p} \nmid \mathrm{a}$ b) $\mathrm{p}|\mathrm{a}$
   c) $\mathrm{a}|\mathrm{b}$ d) $\gcd(\mathrm{a}, \mathrm{b}) = \mathrm{p}$

3) If p is a prime and $k > 0$ then which of the followings are true?
   a) $\varphi(p^k) = p^k - p^{k-1}$ b) $\varphi(p^k) = p^k + p^{k-1}$
   c) $\varphi(p^{k+1}) = p\varphi(p^k)$ d) both a and c

4) The congruence $x \equiv a(mod\ n)$ and $x \equiv b(mod\ m)$ admits a simultaneous solution iff _____.
   a) $\gcd(\mathrm{n}, \mathrm{m})|\mathrm{a} - \mathrm{b}$ b) $\gcd(\mathrm{n}, \mathrm{m}) \nmid \mathrm{a} - \mathrm{b}$
   c) $\gcd(\mathrm{n}, \mathrm{m}) = \mathrm{a}.\mathrm{b}$ d) $\gcd(\mathrm{n}, \mathrm{m}) = 2$

5) If a be a primitive root modulo n and b, k are any integers, then $ind. b^k \equiv$ _____.
   a) $k\ ind. b\ (mod\ \varphi(n))$ b) $k\ ind. b\ (mod\ n)$
   c) $b\ ind. k\ (mod\ n)$ d) $k + ind. b\ (mod\ \varphi(n))$

6) Which of the following is true?
   a) $\varphi(n)$ is always an even number.
   b) $\varphi(n)$ is always an odd number.
   c) $\varphi(n)$ is even for infinitely many values of n.
   d) $\varphi(n)$ is even for only finitely many

7) The general solution of $311x - 112y = 73$ is _____.
   a) $x = 15 - 112t, y = 41 + 311t$
   b) $x = 15 + 112t, y = 41 + 311t$
   c) $x = 41 + 112t, y = 15 + 311t$
   d) $x = 37 + 112t, y = 31 + 311t$

8) The difference of two consecutive cube is _____.
   a) divisible by 2 b) never divisible by 2
   c) Zero d) none

B) Fill in the blanks. **04**

1) The number of integers of $S = \{1,2,3 - - -, n\}$ divisible by a positive integer a is _____.
2) If $a > 1$ and m, n are positive integers then $\gcd(a^m - 1, a^n - 1) =$ _____.
3) The number of integers less than 1896 and relatively prime to 1896 are ___.
4) If 'a' be an integer having order $k(mod\ n)$ and $a \equiv b(mod\ n)$ then the order of $b(mod\ n)$ is _____.

Q.2 Answer the following. (Any Six) (Each 2 Marks) **12**

a) Factorize 2047 using Fermat factorization method.
b) If r is the smallest primitive root of n and $r^h \equiv a(mod\ n)$ then show that $h \equiv ind\ a\ (mod\varphi(n))$.
c) if a and b are any two integers not both zero then show that there exist integers x and y such that $\gcd(a, b) = ax + by$.
d) Find the last two digits of the number 9^{9^9}.
e) If f is multiplicative function and $S(n) = \sum_{d|n} f(d)$ then prove that $S(n)$ is also multiplicative function.
f) Prove that 1729 is absolute pseudo prime.
g) Find the primitive roots of 10.
h) Find $\tau(n)$ and $\sigma(n)$ for $n = 7056$.

Q.3 Answer the following. (Any Three) **12**

a) If the orders of a_1 and a_2 modulo n be k_1 and K_2 respectively and $\gcd(k_1, k_2) = 1$. Then the prove that the order of $a_1a_2(mod\ n)$ is k_1k_2.
b) State and prove Euclid's lemma.
c) Solve $17x \equiv 9\ (mod\ 276)$.
d) Prove that τ and σ are the multiplicative functions.

Q.4 Answer the following. (Any Two) **12**

a) If $\gcd(a, b) = d$ then the equation $ax + by = c$ has a solution iff $d|c$, further if (x_0, y_0) is a solution of $ax + by = c$ then show that all the other solutions are in the form $x_1 = x_0 - \frac{b}{d}t, y_1 = y_0 + \frac{a}{d}t$ for any integer t.
b) State and Prove Wilson's theorem and prove that converse of Wilson's theorem is also true.
c) Show that the integer 2^n has no primitive root for $n \geq 3$.

Q.5 Answer the following. (Any Two) **12**

a) Find the primes not exceeding 140 by using the method Sieve of Eratosthenes.
b) Solve the system of linear congruence's;
$x \equiv 3(mod\ 11), x \equiv 5(mod\ 19), x \equiv 10(mod\ 29)$.
c) State and prove Euler's Theorem.

SLR-HO-7

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Research Methodology in Mathematics (2317103)

Day & Date: Friday, 17-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.8

Q.1 A) Choose correct alternative. **08**

1) Which one is called non-probability sampling?
   a) Quota Sampling b) Cluster Sampling
   c) Systematic sampling d) Stratified random sampling

2) _____ to research is concerned with subjective assessment of attitudes, opinions and behavior
   a) Quantitative approach b) Qualitative approach
   c) Ex post facto approach d) all of these

3) The maximum value of h such that the given author/journal has published at least h papers that have each been cited at least h times is known as _____.
   a) i-10 index b) citation
   c) h-index d) impact factor

4) Science Citation Index was officially launched in 1964 and is now owned by _____.
   a) Google scholar b) Scopus
   c) Clarivate d) UGC CARE

5) The UGC CARE Group 1 includes journals found qualified through ____.
   a) scopus b) web of science
   c) UGC CARE protocols d) SCI

6) Which of the following is not the Method of Research?
   a) Observation b) Historical
   c) Survey d) Philosophical

7) The purpose of _____ is to summarize the contents of the paper.
   a) Introduction b) references
   c) Title d) abstract

8) The longform of SCI is _____.
   a) Science citation index b) Scopus citation index
   c) Science citation India d) Scopus citation India

B) State True/False. **04**

1) Research is an art of scientific investigation.
2) Simulation approach involves the construction of an artificial environment.
3) Deliberate sampling is a kind of probability sampling.
4) Data can be collected through Telephone interview.

Q.2 Answer the following. (Any Six) **12**

a) Write long form of SCI and AMS.
b) Define Citation and impact factor.
c) Give definition of research.
d) Write objectives of research.
e) Explain the need of UGC CARE list.
f) Explain stratified sampling.
g) What is extensive literature survey?
h) Give information about Math Sci Net.

Q.3 Answer the following. (Any Three) **12**

a) State the qualities of good research.
b) Write an expository note on Research Approaches.
c) Give the difference between Research methods and Research Methodology
d) Write note on: The Role of examples in research article.

Q.4 Answer the following. (Any Two) **12**

a) Write the file format of Research article.
b) Explain Do's and Don'ts of Mathematical writing.
c) Give details of how to write abstract and conclusion in research article.

Q.5 Answer the following. (Any Two) **12**

a) Write detail information about different types of sampling.
b) Write the problems encountered by researchers in India.
c) Define and Explain: I10 index, h index, Science citation index

SLR-HO-8

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Number Theory (MSC15108)

Day & Date: Friday, 10-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) If p is prime and $d|p-1$ then the congruence $x^d - 1 \equiv 0\ (mod\ p)$ has _____ solutions.
   a) exactly p b) exactly d
   c) more than d d) pd

2) If $\gcd(a,b) = 1$, then for any integer c, $\gcd(ac,b) =$ _____.
   a) 1 b) $\gcd(a,c)$
   c) $\gcd(ab,c)$ d) $\gcd(b,c)$

3) The congruence $x^2 \equiv -1\ (mod\ p)$, p is a prime, has a solution iff _____.
   a) $p \equiv -1 (mod\ 4)$ b) $p \equiv 0 (mod\ 4)$
   c) $p \equiv 1 (mod\ 4)$ d) $p \equiv 1 (mod\ p^2)$

4) Consider the statements:
   i) If $a^k \equiv b^k (mod\ m)$ then $a \equiv b\ (mod\ m)$ for all $k \geq 1$
   ii) If $a \equiv b\ (mod\ m)$ and $c \equiv d\ (mod\ m)$ then $a + c \equiv b + d\ (mod\ m)$
   a) only i) is true b) only ii) is true
   c) both i) and ii) are true d) both i) and ii) are false

5) The general solution of $311x - 112y = 73$ is _______.
   a) $x = 15 - 112t, y = 41 + 311t$
   b) $x = 15 + 112t, y = 41 + 311t$
   c) $x = 41 + 112t, y = 15 + 311t$
   d) $x = 37 + 112t, y = 31 + 311t$

6) If $p, q_1, q_2, q_3, \ldots, q_n$ are all primes and $p|q_1.q_2.q_3 \ldots\ q_n$, then for $1 \leq k \leq n$
   a) $p \neq q_k$ for all k b) $p|q_k$ for all k
   c) $p = q_k$ for some k d) $q_k|p$ for all k

7) Order of 3 modulo 19 is _____.
   a) 3 b) 19
   c) 18 d) 11

8) The remainder when the sum $S = 1! + 2! + 3! + --- + 999! + 1000!$
   Is divisible by 10.
   a) 7 b) 1
   c) 3 d) 5

9) The smallest divisor other than one of a composite number is _____.
   a) Composite b) Odd
   c) Even d) Prime

10) Solution of $47x \equiv 11\ (mod\ 249)$ is ____.
   a) 49 b) 85
   c) 185 d) 147

B) Write True/False. **06**

1) If $\gcd(m,n) = 1$ where $m > 2, n > 2$ then the integer mn has 2^{mn} primitive roots.
2) If $ca \equiv cb\ (mod\ n)$ and $\gcd(c,n) = 1$ then $a \equiv b\ (mod\ n)$.
3) If a is primitive root modulo n and b, c are any integers, then $ind.(bc) \equiv ind.b + ind.c(mod\ \varphi(n))$.
4) The greatest integer value of $x = -5.9$ is-5.
5) If a and b be the integers not both zero then a and b are relatively prime iff there exist integers x and y such that $ax + by = 1$.
6) If $n = 1$ then $\sum_{d|n} \mu(d) = 0$.

Q.2 Answer the following **16**

a) If $ac \equiv bc\ (mod\ n)$ then show that $a \equiv b\ \left(mod\ \frac{n}{d}\right)$, where $d = gcd(c,n)$.

b) Find the highest power of 17 contained in 30000!

c) If $f(n) = n^2 + 2$ and $n = 6$ then show that $\sum_{d|6} f(d) = \sum_{d|6} F\left(\frac{6}{d}\right)$

d) Define the following terms:
1) Square free integers
2) Linear Congruence

Q.3 Answer the following.

a) If a is a primitive root modulo n and b, c and k are any integers, then prove that, **08**
1) $b \equiv c(mod\ n) \Rightarrow ind\ b \equiv ind\ c\ (mod\ \varphi(n))$
2) $ind.(bc) \equiv ind\ b + ind\ c\ (mod\ \varphi(n))$
3) $ind\ b^k \equiv k\ ind\ b\ (mod\ \varphi(n))$
4) $ind\ 1 \equiv 0\ (mod\ \varphi(n))$

b) Find the general solution of the linear Diophantine equation $39x - 56y = 11$ **08**

Q.4 Answer the following.

a) Write a note on Fermat factorization method and factorize 23247. **08**

b) Solve $49x \equiv 47(mod\ 81)$. **08**

Q.5 Answer the following.

a) State and prove Fundamental theorem of Arithmetic. **10**

b) If f is multiplicative function and $S(n) = \sum_{d|n} f(d)$ then prove that $S(n)$ is also multiplicative function. **06**

Q.6 Answer the following.

a) If p is a prime and $f(x) = a_n x^n + a_{n-1} x^{n-1} + - - - + a_1 x + a_{0,}$ $a_n \not\equiv 0(mod\ p)$ is a polynomial of degree $n \geq 1$ with integral coefficients then show that $f(x) \equiv 0(mod\ p)$ has at least n incongruent solutions $mod\ p$. **10**

b) Show that if one of the two integers $2a + 3b$ or $9a + 5b$ is divisible by 17 then so can the other. **06**

Q.7 Answer the following.

a) If $n = {p_1}^{k_1}{p_2}^{k_2} --- {p_r}^{k_r}$ is a prime factorization of n then prove that, **08**

1) $\tau(n) = (k_1 + 1)(k_2 + 1) --- (k_r + 1)$

2) $\sigma(n) = \left(\frac{{p_1}^{k_1+1} - 1}{p_1 - 1}\right)\left(\frac{{p_2}^{k_2+1} - 1}{p_2 - 1}\right) --- \left(\frac{{p_r}^{k_r+1} - 1}{p_r - 1}\right)$

b) Solve the system of linear congruence's; **08**

$x \equiv 2(mod\ 11), x \equiv 4(mod\ 19), x \equiv 1(mod\ 29)$

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Object Oriented Programming using C++ (MSC15109)

Day & Date: Friday, 10-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) The mechanism of deriving a new class from an old one is called _____.
   a) Abstraction b) Inheritance
   c) Polymorphism d) None of these

2) A ______ is a basic run time entity.
   a) Polymorphism b) Class
   c) Object d) Inheritance

3) ______ is used to declare float data type.
   a) Decimal b) Float
   c) FLOAT d) float

4) ______ is the smallest unit in a program.
   a) Token b) Unit
   c) Abstraction d) Pointer

5) The ______ are explicitly reserved words and cannot be used as names for the program variables or other user-defined program elements.
   a) identifiers b) keywords
   c) string d) operators

6) An ______ function is a function that is expanded in line when it is invoked.
   a) inline b) multiline
   c) pointer d) undefined

7) ______ are operators that are used to format data display.
   a) String b) Identifiers
   c) Keyboards d) Manipulators

8) Identify the incorrect constructor type.
   a) Friend constructor b) Default constructor
   c) Parameterized constructor d) Copy constructor

9) Identify the scope resolution operator.
   a) : b) ::
   c) ?: d) ->

10) The mechanism of giving special meaning to an operator is known as ______ overloading.
   a) function b) pointer
   c) operator d) keywords

B) State True or False. **06**

1) The identifiers refer to the variable name.
2) Object is a basic run time entity.
3) The use of same function name to create functions that perform a variety of different tasks is known as Operator overloading.
4) A derived class with only one base class is called as single inheritance.
5) An object with a constructor can be used as member of union.
6) By default, members of the class are private.

Q.2 Answer the following. **16**

a) What is Operator? Explain different types of operators used in C++.
b) What is Class? Explain the use of class with example.
c) What is default arguments? Explain with example.
d) Explain the basic Data types used in C++.

Q.3 Answer the following. **16**

a) What is Flowchart? Explain different symbols used in flowcharts.
b) Explain the basic concepts of OOP.

Q.4 Answer the following. **16**

a) What is Inheritance? Explain multilevel Inheritance with suitable example.
b) What is friend function? Explain the importance of friend function.

Q.5 Answer the following. **16**

a) What is array? Explain Two dimensional array with example.
b) What is constructor? Explain the use of copy constructor.

Q.6 Answer the following. **16**

a) Explain different types of memory management operators in C++.
b) Write a C++ program to implement Operator overloading (Assume your own data).

Q.7 Answer the following. **16**

a) Explain the use of call by reference with suitable example.
b) Write a C++ program to implement single inheritance. (Assume your own data).

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Algebra - I (MSC15101)

Day & Date: Monday, 13-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If G is abelian, then its commutator subgroup $G' =$ _______.
   a) ϕ
   b) $\{e\}$
   c) G
   d) some proper non-trivial subgroup of G

2) If G is a group of order 5, then its class equation is ______.
   a) $5 = 3 + 2$
   b) $5 = 1 + 2 + 2$
   c) $5 = 1 + 1 + 1 + 1 + 1$
   d) $5 = 3 + 1 + 1$

3) If G is an abelian group, then G is ______.
   a) solvable
   b) nilpotent
   c) both solvable and nilpotent
   d) None of these

4) A polynomial $f(x) = x^2 - 2$ is ______.
   a) irreducible over $\mathbb{Q}$
   b) reducible over $\mathbb{Q}$
   c) irreducible over $\mathbb{R}$
   d) irreducible over $\mathbb{C}$

5) Which of the following is/are zeros of $f(x) = x^2 + x + 1$ in $\mathbb{Z}_3$?
   a) 0
   b) 1
   c) 2
   d) None of these

6) If D is a Euclidean domain, then D is ______.
   a) a principal ideal domain
   b) Integral domain
   c) an unique factorization domain
   d) All of the above

7) If D is a field, then $D[x]$ is _______.
   a) a field
   b) integral domain
   c) commutative ring without unity
   d) non-commutative ring with unity

8) If G is a group then $\frac{G}{G'}$ is ______.
   a) cyclic
   b) abelian
   c) non-cyclic
   d) non-abelian

9) Consider the two statements.
Statement P: Every normal series is subnormal
Statement Q: Every composition series is principal series.
Then
a) Only P is true
b) Only Q is true
c) Both P and Q are true
d) Both P and Q are false

10) If D is a PID and p an element of D then ______.
a) p is irreducible $\Rightarrow < p >$ is maximal
b) $< p >$ is maximal $\Rightarrow p$ is irreducible
c) both (a) and (b) hold
d) neither (a) nor (b) hold

B) Fill in the blanks. **06**

1) If G is a group of order p^3 (p is a prime number), then $Z(G)$ has exactly p elements.
2) If D is not a PID, then it is not ED.
3) If F is a field then $a \in F$ zero of a non-constant polynomial $f(x)$, then $(x-a)|f(x)$.
4) If G is a group and X is a G-set, then $X_g, g \in G$ is a subgroup of X.
5) If G is a group of order 12, then order of Sylow-2-subgroup of G is 4.
6) A ring $< 2\mathbb{Z}, +, \cdot >$ is not an integral domain.

Q.2 Answer the following. **16**

a) If G is a group then prove that its commutator subgroup G' is normal subgroup of G.
b) Define subnormal series. Show that a subnormal series need not be normal.
c) Define unique factorization domain.
d) If $p_1(x), p_2(x), \dots, p_n(x)$ are n primitive polynomials in a ring $D[x]$, then prove that $p_1(x)p_2(x)\dots p_n(x)$ is also a primitive polynomial.

Q.3 Answer the following.

a) If $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = x^2 + 2x - 3$ in $\mathbb{Z}_7[x]$, then find $q(x), r(x)$ in $\mathbb{Z}_7[x]$ such that $f(x) = g(x)q(x) + r(x)$, with deg $r(x) < 2$. **08**
b) If G is a group and X is a G-set, then prove that $\sum_{g\in G} |X_g| = r|G|$, where r is the number of orbits in X under G. **08**

Q.4 Answer the following.

a) Prove that any two subnormal series for a group G have isomorphic refinements. **10**
b) If D is a PID and a, b are nonzero elements of D, then prove that there exists a gcd of $a, b.$ Furthermore, prove that each gcd of a and b can be expressed in the form $\lambda a + \mu b$ for some $\lambda, \mu \in D$. **06**

Q.5 Answer the following.

a) State and prove Cauchy's theorem. **08**
b) Show that the following polynomials in $\mathbb{Z}[x]$ are irreducible over $\mathbb{Q}$ using Eisenstein's criteria. **08**
i) $x^2 - 12$
ii) $8x^3 + 6x^2 - 9x + 24$

Q.6 Answer the following.

a) Find the isomorphic refinements of the series: **10**
$\{0\} < 72\mathbb{Z} < 24\mathbb{Z} < 4\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 36\mathbb{Z} < 9\mathbb{Z} < \mathbb{Z}$.

b) Prove that $\mathbb{Z}$ has no principal series. **06**

Q.7 Answer the following.

a) Prove that no group of order 96 is simple. **08**

b) If $\mathbb{F}$ is a field, then prove that an ideal $< p(x) > \neq \{0\}$ of $\mathbb{F}[x]$ is maximal iff $p(x)$ is irreducible over $\mathbb{F}$. **08**

SLR-HO-11

Seat No.	

Set P

M.Sc. (Semester -I) (Old) (CBCS) Examination: March/April - 2024
MATHEMATICS
Real Analysis - I (MSC15102)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The supremum of set of all lower sums is called ______.
   a) Upper integral b) Lower integral
   c) Both a and b d) None of these

2) If f have local extrema at C then, $f'(C)$ = _____.
   a) 1 b) -1
   c) 0 d) constant

3) Consider the following statements:
   I) Function having only one point discontinuity is integrable.
   II) Function having finite no. of points of discontinuity is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

4) For any partition P, the norm of partition is defined as $\mu(p) =$ _____.
   a) $maxP$ b) $minP$
   c) $min\,\Delta\, x_i$ d) $max\,\Delta\, x_i$

5) By first mean value theorem, if a function f is continuous on $[a,b]$ then there exist a number ξ in $[a,b]$ such that $\int_a^b f(x)dx =$ _____.
   a) $f(\xi)(a-b)$ b) $f(\xi)(b-a)$
   c) $f(\xi)(a+b)$ d) $f'(\xi)(a-b)$

6) The directional derivative of $f(x,y) = xy$ at point $(1,1)$ in the direction $(1,0)$ is ______.
   a) 1 b) $(1,1)$
   c) y d) x

7) A necessary and sufficient condition for integrability of a bounded function is ______.
   a) $lim_{\mu(P)\to\infty}\big(U(P,f) - L(P,f)\big) = 0$
   b) $lim_{\mu(P)\to\infty}\big(U(P,f) + L(P,f)\big) = 0$
   c) $lim_{\mu(P)\to 0}\big(U(P,f) + L(P,f)\big) = 0$
   d) $lim_{\mu(P)\to 0}\big(U(P,f) - L(P,f)\big) = 0$

8) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is given by $P^* =$______.
   a) $P_1 \cap P_2$ b) $P_1 + P_2$
   c) $P_1 - P_2$ d) $P_1 \cup P_2$

9) The length of subinterval $[x_{i-1}, x_i]$ is given by $\Delta x_i =$ ______.
   a) $x_{i-1} - x_i$ b) $x_i - x_{i-1}$
   c) $x_i + x_{i-1}$ d) None of the above

10) If f and g are integrable functions then _____ is also integrable.
   a) $f + g$ b) $f - g$
   c) $f.g$ d) all of the above

B) Fill in the blanks. **06**

1) Riemann - Stieltje's integral reduces to Riemann integral if $\alpha(x) =$ _____.
2) The Riemann Sum is given by $S(P, f) =$ ______.
3) The mean value of $\int_0^1 x^2 dx$ in $[0,1]$ is ______.
4) The condition of ______ is necessary for a function to assume its mean value ξ in given interval by first mean value theorem.
5) If $f(x) = x$ on $[0,1]$ and divide the interval into two equal sub intervals then $L(P, f) =$ ______.
6) A bounded function f is integrable on $[a, b]$ if the set of points of discontinuity has ______ limit points.

Q.2 Answer the following. **16**

a) Define: Upper Integral and Lower Integral.
b) Write Mean Value theorem for the functions f from $R^n \to R$.
c) If $\int_{-1}^{2} x^2 dx = 3$ then find its mean value.
d) Examine whether the function $f(x) = x^2 + 4x + 3$ on $[-10, 10]$ have local extrema or not.

Q.3 Answer the following.

a) If a function f is continuous on $[a, b]$ then prove that there exists a number ξ in $[a, b]$ such that $\int_a^b f(x)dx = f(\xi)(b - a)$. **08**
b) If a function f is bounded and integrable on $[a, b]$ then prove that the function F defined as, $F(x) = \int_a^x f(t)dt; a \leq x \leq b$ is continuous on $[a, b]$. Furthermore if f is continuous at a point c of $[a, b]$ then prove that F is derivable at c and $F'(c) = f(c)$. **08**

Q.4 Answer the following.

a) If f_1 *and* f_2 are two bounded and integrable functions on $[a, b]$ then prove that $f_1 + f_2$ is also integrable on $[a, b]$ and also prove that $\int_a^b (f_1 + f_2)dx = \int_a^b f_1 dx + \int_a^b f_2 dx$ **08**
b) If f is differentiable function at c with total derivative T_c then prove that the directional derivative $f'(c; u)$ exists for every u in R^n and also prove that $T_c(u) = f'(c; u)$ **08**

Q.5 Answer the following.

a) Solve. $\int_0^5 (4x+5)dx$ **10**

b) If f is bounded function on $[a,b]$ then prove that for every $\in > 0$ there corresponds $\delta > 0$ such that **06**

1) $U(P,f) < \int_a^{\bar{b}} f(x)dx + \in$

2) $L(P,f) > \int_{\underline{a}}^{b} f(x)dx - \in$

for every partition P of $[a,b]$ with norm $\mu(P) < \delta$

Q.6 Answer the following.

a) Find directional derivative of **08**

$$f(x) = \begin{cases} \frac{x^2.y}{x^4+y^2} & if \quad (x,y) \neq (0,0) \\ 0, & if \quad (x,y) = (0,0) \end{cases}$$

b) Prove that: The oscillation of a bounded function f on an interval $[a,b]$ is the supremum of the set $\{|f(x_1) - f(x_2)|/x_1, x_2 \in [a,b]\}$ of numbers. **08**

Q.7 Answer the following.

a) Prove that: A necessary and sufficient condition for the integrability of a bounded function f is that for every $\in > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a,b]$ with norm $\mu(P) < \delta, U(P.f) - L(P,f) < \in$ **08**

b) Prove that: Every continuous function is integrable. **08**

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Differential Equations (MSC15103)

Day & Date: Friday, 17-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The wronskian of f and g is $3e^{4x}$ and $f(x) = e^{2x}$ then the differential equation for $g(x)$ is _____.
   a) $2g'(x) - g(x) = e^{-x}$ b) $2g'(x) - g(x) = e^{x}$
   c) $g'(x) - 2g(x) = 3e^{2x}$ d) None of these

2) If $\alpha \pm \beta i$ are two complex conjugate roots of characteristic equation, then two solutions are given by _____.
   a) $\sin\beta x, \cos\alpha x$ b) $\sin\alpha x, \cos\beta x$
   c) $e^{\alpha x}, e^{\beta x}$ d) $e^{\alpha x}\sin\beta x, e^{\alpha x}, \cos\beta x$

3) $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$ is ______.
   a) Euler equation b) Bessel equation
   c) Legendre's equation d) Wave equation

4) The expansion of $\sin x$ is _____.
   a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ b) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$
   c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ d) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$

5) If P is polynomial such that deg $(P) = n$ and $P(z) = (z-a).q(z)$ then q has _____ roots.
   a) n b) $n-1$
   c) $n+1$ d) 0

6) Two solutions of differential equation are always _____.
   a) Linearly dependent b) Linearly independent
   c) Both a and b d) None of these

7) A function φ is solution of initial value problem $y' = f(x,y), y(x_0) = y_0$ on an interval I iff it is solution of the integral equation _____.
   a) $y = y_0 + \int_{x_0}^{x} f(t,\varphi(t))dt$ b) $y = y_0$
   c) $y = \int_{x_0}^{x} f(t,\varphi(t))dt$ d) $y = y_0 - \int_{x_0}^{x} f(t,\varphi(t))dt$

8) solutions of $y'' - 2y' - 3y = 0$ are _____.

a) e^{3x}, e^{x} b) e^{2x}, e^{-3x}

c) e^{3x}, e^{-x} d) $2x, 3x$

9) $\frac{d}{dx}\left[J_{\frac{1}{2}}(x)\right] =$ _____.

a) $\sin x$ b) $\cos x$

c) $\sqrt{\frac{2}{\pi x}}\sin x$ d) $\sqrt{\frac{2}{\pi x}}\cos x$

10) The Lipschitz constant for the function $f(x,y) = 4x^2 + y^2$ on $S = \{(x,y) || x| \le 1, |y| \le 1\}$ is _____.

a) $k = 1$ b) $k = 2$

c) $k = 0$ d) $k = \infty$

B) Write true/false. **06**

1) The value of Wronskian $W(x, x^2, x^3)$ is $2x^2$.
2) A singular point which is not regular is called irregular singular point.
3) The order of differential equation whose solutions are $\sin x$ and $\cos x$ is 4.
4) If r_1 is root of multiplicity m of characteristic polynomial $P(r)$ of n^{th} order linear differential equation with constant coefficients then $P(r_1) = 0, P'(r_1) = 0, \dots, P^{m-1}(r_1) = 0$
5) With usual notations, $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.
6) The equation of the form $y'' + a_1 y' + a_2 y = 0$ Where a_1 and a_2 are complex constant and is called First order non-homogeneous differential equation.

Q.2 Answer the following. **16**

a) Compute the first four successive approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$ of the initial value problem $y' = 1 + xy, y(0) = 1$.

b) If φ_1 and φ_2 are two solutions of second order differential equations with constant coefficient $L(y) = y'' + a_1 y' + a_2 y = 0$ then show that their linear combination is also a solution of $L(y) = y'' + a_1 y' + a_2 y = 0$

c) show that $xJ_n{}'(x) = xJ_{n-1}(x) - nJ_n(x)$.

d) Solve, $y'' + (3i - 1)y' - 3iy = 0$

Q.3 Answer the following.

a) Prove that two solutions φ_1, and φ_2 of $L(y) = 0$ are linearly independent on any interval I if $W(\varphi_1, \varphi_2)(x) \ne 0$. **08**

b) Prove that $\int_{-1}^{1} p_m(x) p_n(x) dx = 0$ if $m \ne n$. **08**

Q.4 Answer the following.

a) Show that for any real number x_0 and constant α, β there exists a solution φ of the initial value problem $L(y) = 0$ and $\varphi(x_0) = \alpha$ and $\varphi'(x_0) = \beta$. **08**

b) Derive Bessel function of zero order of first kind. **08**

Q.5 Answer the following.

a) Show that $f(x, y) = 4x^2 + y^2$ satisfies Lipschitz condition on the set $S: |x| \leq 1, |y| < \infty$. **08**

b) Prove that the wronskian of two solutions of equation $y'' + a_1y' + a_2y = 0$ is either identically zero or never zero on (a, b). **08**

Q.6 Answer the following.

a) Prove that a function φ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I iff it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f\big(t, \varphi(t)\big)dt$. **10**

b) Show that infinity is not a regular singular point for the equation $y'' + ay' + by = 0$ where a, b are constants, not both zero. **06**

Q.7 Answer the following.

a) Solve, $4y'' - y = e^x$. **08**

b) Find the general solution of $y'' + y = \tan x, \frac{-\pi}{2} < x < \frac{\pi}{2}$ **08**

SLR-HO-13

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April - 2024
MATHEMATICS
Classical Mechanics (MSC15104)

Day & Date: Monday, 20-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Determinant value of an orthogonal matrix is _____.
   a) 1 b) -1
   c) either 1 or -1 d) neither 1 nor -1

2) Which of the following does not represents a rotation?
   a) orthogonal matrix with determinant -1
   b) orthogonal matrix with determinant +1
   c) Eulerian angles
   d) Both b and c

3) Rheonomic constraint depends on ______.
   a) co-ordinates b) time
   c) momentum d) both a and b

4) Geodesic on the surface of sphere is ______.
   a) parabola b) cycloid
   c) hyperbola d) arc of great circle

5) Hamiltonian H is independent of ______.
   a) generalized coordinates b) generalized velocity
   c) generalize momentum d) time

6) Number of Cartesian coordinates require to describe configuration of double pendulum is/are ______.
   a) 1 b) 2
   c) 3 d) 4

7) Lagrangian is defined as ______.
   a) L = T-V b) L = T+V
   c) 2T+V d) L = 2T-V

8) Brachistochrone problem deals with ______.
   a) a curve with extremum length
   b) a curve with extremum area
   c) a curve with extremum volume
   d) a curve with extremum time

9) Newton's equation of motion can be derived from Lagrange's equation.
   a) true b) false
   c) can't say d) may be

10) Conservative force is only depends on ______.
   a) time b) velocity
   c) co-ordinates d) both (a) and (b)

B) Fill in the blanks. **06**

1) Euler - Lagrange's differential equations are _____ conditions for extremum of a functional.
2) Shortest distance between any two points is a _____.
3) Bead sliding in moving wire is ______ constraints.
4) Gravitational force is an example of ______.
5) The extremum of the functional J[y(x)] is called local maximum if ΔJ _____.
6) The curve is ______ for which area of surface of revolution is mini-mum when revolved about y-axis.

Q.2 Answer the following. **16**

a) State modified Hamilton's principle.
b) Define Degrees of freedom and Generalised co-ordinates and give one example each.
c) Explain four types of constraints.
d) Show that: The generalised momentum corresponding to cyclic co-ordinates is conserved.

Q.3 Answer the following. **16**

a) Obtain Lagranges's equation of motion for simple pendulum.
b) Establish the relation between δ - *variation* and Δ - *variation.*

Q.4 Answer the following. **16**

a) Prove that: In case of orthogonal transformation the inverse matrix is identified by its transpose. i.e. $A^{-1} = A^T$
b) Derive the equation of motion of Atwood's machine. **08**

Q.5 Answer the following.

a) Show that: The shortest distance between two points in a plane is a straight line. **10**
b) State and prove Hamilton's principle by using Lagranges's equation. **06**

Q.6 Answer the following.

a) Derive Newton's equation of motion from Lagrange's equation of motion. **08**
b) A particle of mass m moving in a plane under the action of an inverse square law of attractive force. Derive the Lagrangian L and hence equation of its motion. **08**

Q.7 Answer the following.

a) Find Euler-Lagrange's differential equation satisfied by $y(x)$ for which the integral **08**
$I = \int_{x_1}^{x_2} f(y, y', x)dx$ has extremum value, where $y(x)$ is twice differentiable function satisfying $y(x_1) = y_1 \; and \; y(x_2) = y_2$
b) Find the extremal of the function $I(y(x)) = \int_{x_0}^{x_1} (16y^2 - (y'')^2 + x^2)dx$. **08**

SLR-HO-14

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Field Extension Theory (2317201)

Day & Date: Thursday, 09-05-2024 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to the right indicates full marks.

Q.1 A) Choose the correct alternative **08**

1) For every prime p and every positive integer m there exist a finite field with _____ elements.
   a) m^p b) p^m
   c) $m.p$ d) None of these

2) For a field of characteristic zero _____.
   I) Every finite extension is simple extension
   II) Every finite extension is separable extension
   a) only I is true b) Only II is true
   c) Both are true d) Both are false

3) If $\sqrt{7}$ and $\sqrt{5}$ are constructible numbers then _____ is also constructible.
   a) $7^{3/2}$ b) $5^{11/2}$
   c) $\sqrt{7}+\sqrt{5}$ d) All of these

4) The complex zeros of a polynomial with ______ coefficients occur in conjugate pairs.
   a) real b) integer
   c) rational d) complex

5) The set of all ______ forms a field.
   a) integers b) transcendental numbers
   c) irrational d) algebraic numbers

6) The group $G(Q(\sqrt{2}), Q)$ has _____ elements.
   a) 2 b) 1
   c) 3 d) finite

7) If a & b are algebraic over F of degree m & n respectively then $a+b$ is algebraic of degree _____.
   a) $m+n$ b) mn
   c) atmost mn d) atmost $m+n$

8) If K is an extension of F and every element in K which is outside F is moved by some element in $G(K, F,)$ then K is _____ extension.
   a) finite b) simple
   c) separable d) normal

B) State whether following statements are true or false. **04**

1) It is impossible to square any circle of constructible radius by straight edge and compass.
2) $\sqrt{2} \in R$ is algebraic degree 1 over R
3) A field of complex numbers is a normal extension of field of real numbers.
4) Transitivity of algebraic extension is not always true.

Q.2 Answer the following. (Any Six) **12**

a) Define
 1) Field Extension
 2) Algebraic element

b) If a and b are constructible numbers then prove that $a+b, a-b$ are also constructible.

c) Check whether the following numbers is algebraic or transcendental over given fields. If algebraic find the degree.
 1) $\sqrt{\pi}$ over R
 2) $\sqrt{3}+i$ over Q

d) Find the fixed field of $G\big(Q(\sqrt{7}), Q\big)$.

e) Construct a field with 9 elements

f) Prove that R is not normal extension of Q.

g) Define:
 1) Separable element
 2) Perfect field

h) Find degree and basis of $Q(2^{1/3})$ over Q.

Q.3 Answer the following. (Any Three) **12**

a) Find splitting field of x^4+4 over Q.

b) Write short note on elementary symmetric functions.

c) With usual notations Prove or disprove that: $Q(\sqrt{3}+\sqrt{7}) = Q(\sqrt{3}, \sqrt{7})$

d) Prove that: If $a \in K$ be algebraic over F and $p(x)$ be minmal polynomial for a over F then prove that $p(x)$ is irreducible over F.

Q.4 Answer the following. (Any Two) **12**

a) If $a \in K$ be algebraic over F then prove that any two minimal monic polynomial for a over F are equal.

b) Prove that: A field K is normal extension of a field F of characteristic zero iff K is splitting field of some polynomial over F.

c) Prove that: The Galois group of a polynomial over a field F of characteristic zero is isomorphic to a group of permutation of its roots.

Q.5 Answer the following. (Any Two) **12**

a) If K is a finite extension of a field F then prove that $G(K,F)$ is a finite group and its order $O(G(K,F)$ statisfies the relation $O(G(K,F) \leq [K:F]$.

b) If K is an extension of field F then prove that the set of all algebraic elements of K over F forms a subfield of K.

c) If K be an extension of F and $a \in K$ be algebraic over F then prove that $F(a)$ is isomorphic to $\frac{F[x]}{V}$ where V is and ideal of $F[x]$ generated by the minimal polynomial for a over F.

SLR-HO-15

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April – 2024
MATHEMATICS
General Topology (2317202)

Day & Date: Saturday, 11-05-2024 — Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **08**

1) In a T-space $< X, \Im >$, which of the following is always true?
   a) $i(E) \subset E$ b) $E \subset i(E)$
   c) $i(E) = E$ d) $E' = i(E)$

2) In a T-space $< X, \Im >, c(E)$ is ______.
   a) the smallest closed set contained in E
   b) the smallest closed set containing E
   c) the largest open set contained in E
   d) the largest open set containing E

3) If $< X, \Im >, < X^*, \Im^* >$ are two T-spaces and if $f: X \to X^*$ is a function, then f is continuous at $x \in X$ if ______.
   a) $< X, \Im >$ is indiscrete T-space
   b) $< X^*, \Im^* >$ is discrete T-space
   c) $\{X\} \in \Im$
   d) None of the above

4) In a discrete T-space $< X, \Im >$ every subset of X is ______.
   a) an open set b) a closed set
   c) both open and closed d) neither open nor closed

5) A co-finite topology on a finite set X reduces to ______.
   a) discrete topology b) indiscrete topology
   c) co-countable topology d) usual topology

6) In any T-space $< X, \Im >$, consider the following two statements
   P: ϕ is always open and closed Q: X is both open and closed. Then,
   a) only statement P is true b) only statement Q is true
   c) both P and Q are true d) both P and Q are false

7) In a discrete T-space $< X, \Im >$, the only connected sets are ______.
   a) empty set b) all open sets
   c) all closed sets d) all singleton sets

8) Any co-countable space is a _______.
   a) T_0 space b) connected space
   c) compact space d) all of the above

B) State whether True or False. **04**

1) In any co-countable T-space $< X, \mathfrak{I} >$, a subset A of X is open iff $X - A$ is finite.
2) In usual T-space $< \mathbb{R}, \mathfrak{I}_u >$, every open interval (a, b) is an open set.
3) Every indiscrete T-space is a connected space.
4) Usual T-space $< \mathbb{R}, \mathfrak{I}_u >$, is a compact space.

Q.2 Answer any six of the following. **12**

a) Define a continuous function on a topological space.
b) Define compact space.
c) Define homeomorphism.
d) Prove that in any T-space $< X, \mathfrak{I} >$, prove that $A \subset B \Rightarrow d(A) \subset d(B)$ where $A, B \subset X$.
e) If $< X, \mathfrak{I} >, < X, \mathfrak{I}^* >$ are two T-spaces and if $i: X \to X$ is an identity function. If $\mathfrak{I}^* \leq \mathfrak{I}$, then prove that i is a continuous function.
f) Define separated sets and connected space.
g) Prove that any metric space is a T_2 space.
h) Define first axiom space.

Q.3 Answer any three of the following. **12**

a) If any T-space $< X, \mathfrak{I} >$, prove that $d(A \cup B) = d(A) \cup d(B)$ for any $A, B \subset X$.
b) If any T-space $< X, \mathfrak{I} >$, prove that $i(E) = E'^{-'}$ where $E \subset X$.
c) Prove that co-finite topological space is a compact space.
d) Prove that being a T_1 space is a topological property.

Q.4 Answer any two of the following. **12**

a) Prove that a T-space $< X, \mathfrak{I} >$, is compact if every family of closed sets having a finite intersection property has non-empty intersection.
b) If $< X, \mathfrak{I} >$, is a T-space and C is a connected set having non-empty intersection with both a set E and the complement of E, then prove that C has a non-empty intersection with the boundary of E.
c) Define regular space. Prove that being a regular space is a hereditary property.

Q.5 Answer any two of the following. **12**

a) If $X = \{a, b, c, d\}, \mathfrak{I} = \{\phi, \{a, d\}, \{a, b\}, \{a, b, d\}, X\}$ then find the derived set of $\{b, c, d\}$. Also find interior of $\{a, b, c\}$.
b) Prove that A topological space X is normal iff for any closed set F and an open set G containing F, there exists an open set H such that $F \subseteq H \subseteq \overline{H} \subseteq G$.
c) In any T-space $< X, \mathfrak{I} >$, prove that for any subset A of $X, \bar{A} = A \cup d(A)$.

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April-2024
MATHEMATICS
Complex Analysis (2317207)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) If $f: C \to C$ defined by $f(z) = z^3 + z$ is an analytic function then the zeros of the function f is _____.
   a) $0,1,-1$ b) $0,i,-i$
   c) $0,2,-1$ d) $1,i,-i$

2) Which of the following mappings does not changes the size and shape of the figure?
   a) $S(z) = z + \beta, T(z) = ze^{i\theta}$ b) $S(z) = z + \beta, T(z) = az; a > 1$
   c) $S(z) = ze^{i\theta}, T(z) = 5z$ d) $S(z) = \frac{1}{z}, T(z) = 5z$

3) The function $f(z) = \cot z$ is/has _____.
   a) singularities at $z = \pm\frac{(n+1)\pi}{2}$ b) singularities at $z = n\pi$
   c) analytic only at $z = 0$ d) analytic everywhere

4) If f have an isolated singularity at $z = 0$ and $f(z) = \sum_{n=-\infty}^{\infty} a_n(-a)^n$ is its Laurent expansion about $z = 0$ then the residue of f at $z = a$ is _____.
   a) a_{-1} b) a_0
   c) a_{-2} d) a_1

5) The radius of convergence of the power series $\sum_{n=0}^{\infty} z^{5n}$ is _____.
   a) 0 b) 1
   c) ∞ d) $\sqrt{2}$

6) A function which has poles as its only singularities in the finite part of the plane is said to be a _____.
   a) Analytic function b) Entire function
   c) Meromorphic function d) Harmonic function

7) The residue of the function $f(z) = \frac{\sin z}{z}$ at $z = 0$ is _____.
   a) $\frac{1}{4!}$ b) $\frac{-1}{7!}$
   c) 1 d) 0

8) If image of an open set is not open under an analytic function then the function is _____.
   a) not analytic b) constant
   c) non-constant d) not differentiable

B) State True/False. **04**

1) If pole of the bilinear transformation lies on the boundary then the image is Circle.
2) A shape with three or more than three sides is called triangular path.
3) If $T(z) = \frac{z-5}{z}$ then $T^{-1}(z) = \frac{z+5}{-z}$
4) $F(z) = \frac{1}{z-4}$ then $f(z)$ has non-isolated singularity at $z = 0$.

Q.2 Answer the following. (Any Six) **12**

a) Prove that $\int_{|z|=1} ze^{\frac{1}{z}} = \pi i$.

b) Find residue of $f(z) = \frac{z^2+1}{(z+2)(z-1)^1} dz$ all singularities of f.

c) Define isolated and non-isolated singularity.

d) Show that the Mobius transformation is the composition of translation, dilation and inversion.

e) Find all the zeros of $f(z) = \sin z$ and $g(z) = \cos z$.

f) Illustrate the construction of cross ratio.

g) Define fixed point and critical point.

h) Evaluate $\int_{|z|=\frac{3}{2}} \tan \pi z \, dz$

Q.3 Answer the following. (Any Three) **12**

a) Show that the set of all bilinear transformation forms a non-abelian group under composition.

b) If $\gamma: [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that, $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z-a}$ is an integer.

c) State and prove Argument Principle.

d) Prove that every non-constant polynomial has a root in complex plane.

Q.4 Answer the following. (Any Two) **12**

a) If f is analytic in $B(a, R)$ then prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n; |z-a| < R$ where, $a_n = \frac{1}{n!} f^n(a)$ and the series has radius of convergence $\geq R$.

b) State and prove Casorati Weierstrass theorem.

c) Show that $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}; |a| > |b|$ & a and b are real number.

Q.5 Answer the following. (Any Two) **12**

a) If G be an open subset of the complex plane C and $f: G \to C$ be an analytic function. If γ is a closed rectifiable curve in G such that, $\eta(\gamma; w) = 0; \forall\, w \in C - G$. Then for $a \in G - \{\gamma\}$ prove that,

$$f(a).\eta(\gamma; a) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w-a} dw$$

b) Find Laurent series expansion of $\frac{1}{z(z-1)(z-2)}$

i) $ann(0; 0,1)$
ii) $ann(0; 1,2)$

c) If z_1, z_2, z_3, z_4 be the four distinct points in C_∞ then prove that the cross ratio (z_1, z_2, z_3, z_4) is real iff all four points lie on a circle or straight line.

SLR-HO-20

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Algebra II (MSC15201)

Day & Date: Thursday, 09-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternatives. **10**

1) The order of $\frac{Z_3[x]}{<x^2+1>}$ is _____.
   a) 3 b) 9
   c) 8 d) 5

2) If characteristic of F is zero and $f(x) \in F[x]$ is irreducible then $f(x)$ has______ roots.
   a) multiple b) distinct
   c) imaginary d) real

3) Every finite extension is a simple extension. This statement is true for a field of characteristic ______.
   a) finite b) prime
   c) nonprime d) zero

4) The fixed field of $G(K, F)$ is ______ F.
   a) contained in b) contains
   c) subfield of d) equal to

5) The element $\sqrt{1+\sqrt{3}}$ is algebraic over Q of degree ______.
   a) 4 b) 2
   c) 8 d) 3

6) The group $G\left(Q(\sqrt{2}), Q\right)$ has ______ elements.
   a) 2 b) 1
   c) 3 d) finite

7) The number π is algebraic over ______.
   a) R b) Q
   c) $Q(i)$ d) $Q(\sqrt{2})$

8) The splitting field of x^2-1 over Q is _____.
   a) $Q(i)$ b) R
   c) Q d) C

9) Which of the following is/are true?
   I) A set of rational numbers is a subfield of R.
   II) A set of irrational numbers is a subfield of R.
   a) Only II is true b) Only I is true
   c) Both are true d) Both are false

10) The field F of all constructible real numbers contains ______.

a) R b) C

c) Q d) All of these

B) State True or False. **06**

1) π is algebraic over R.
2) The field C of complex numbers is simple extension of R the field of real numbers.
3) There exists a field with 10 elements.
4) The set of all constructible numbers forms a subfield of field of Real numbers.
5) If a^m=e then 'a' has order m.
6) Every rational number is left fixed by any automorphism on any extension field K.

Q.2 Answer the following. **16**

a) Check whether $3+\sqrt{7}$ is algebraic over Q or not.

b) Prove that: Every finite extension is algebraic extension.

c) Find degree and basis of $Q(2^{1/4}, i)$ over Q.

d) Write short note on Constructible numbers.

Q.3 Answer the following.

a) Let F be a field & $f(x) \in F[x]$ be such that $f'(x)=0$ then prove that **08**

1) If the characteristic of $F=0$ then $f(x)=a \in F$.
2) If the characteristic of $F=p\neq 0$ then $f(x)=g(x^p)$ for some polynomial $g(x) \in F[x]$.

b) If K is a finite extension of field F then Prove that $G(K,F)$ is a finite group and it satisfies the relation $O(G(K,F)) \leq [K:F]$. **08**

Q.4 Answer the following.

a) If a and b in field K are algebraic over field F of degrees m and n respectively then prove that $a+b$, $a-b$, ab and $a/b(b\neq 0)$ are algebraic over F of degrees atmost mn. **08**

b) If $p(x)$ is an irreducible polynomial in $F[x]$ of degree $n \geq 1$ then prove that there is an extension E of F such that $[E:F]=n$ in which $p(x)$ has a root. **08**

Q.5 Answer the following.

a) If α be zero of a polynomial $p(x)=x^2+x+1 \epsilon Z_2[x]$ is irreducible over Z_2, then find $Z_2(\alpha)$ and its addition and multiplication tables. **08**

b) Prove that: Any two finite fields having the same number of elements are isomorphic. **08**

Q.6 Answer the following.

a) With usual notations Prove or disprove that: $Q(\sqrt{3}+\sqrt{7}) = Q(\sqrt{3},\sqrt{7})$ **08**

b) Prove that: The polynomial $f(x) \in F[x]$ has a multiple root iff $f(x)$ & $f(x)$ have a non-trivial common factor. **08**

Q.7 Answer the following.

a) If $a \in K$ be algebraic over F then prove that any two minimal monic polynomial for a over F are equal. **08**

b) Prove that: Any finite extension of a field of characteristic zero is a simple extension. **08**

SLR-HO-21

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
Real Analysis - II (MSC15202)

Day & Date: Saturday, 11-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) A set $E \subset R$ is called measurable if for any subset A of R, _____.
   a) $m^*(A) = m^*(A \cap E) + m^*(A \cap \bar{E})$
   b) $m^*(A) \neq m^*(A \cap E) + m^*(A \cap \bar{E})$
   c) $m^*(A) \leq m^*(A \cap E) + m^*(A \cap \bar{E})$
   d) $m^*(A) = 0$

2) If f is any function then, $D\_f(x) =$ _____.
   a) $\overline{\lim_{h\to 0^+}} \frac{f(x+h)-f(x)}{h}$ b) $\overline{\lim_{h\to 0^+}} \frac{f(x)-f(x-h)}{h}$
   c) $\underline{\lim_{h\to 0^+}} \frac{f(x)-f(x-h)}{h}$ d) $\underline{\lim_{h\to 0^+}} \frac{f(x+h)-f(x)}{h}$

3) If P is a non-measurable set then χ_P is _____.
   a) measurable function b) non-measurable function
   c) step function d) simple function

4) If E be a measurable subset of set of all real numbers then _____.
   a) E^c may not be measurable b) E^c is closed
   c) E^c measurable d) E^c is open

5) If f is a function of bounded variation on $[a, b]$, then _____.
   a) $T_a^b(f) = p_a^b(f) - N_a^b(f)$
   b) $p_a^b(f) - N_a^b(f) = f(b) - (f)(a)$
   c) $T_a^b(f) = p_a^b(f) \times N_a^b(f)$
   d) $p_a^b(f) - N_a^b(f) = f(b) + (f)(a)$

6) A function φ defined on an open interval (a, b) is convex if for each $x, y \in (a, b)$ and each $\lambda, 0 \leq \lambda \leq 1$, we have _____.
   a) $\varphi(\lambda x + (1-\lambda)y) = \lambda\varphi(x) + (1-\lambda)\varphi(y)$
   b) $\varphi(\lambda x + (1-\lambda)y) \leq \lambda\varphi(x) + (1-\lambda)\varphi(y)$
   c) $\varphi(\lambda x + (1-\lambda)y) \geq \lambda\varphi(x) + (1-\lambda)\varphi(y)$
   d) None of these

7) If f and g are simple functions then _____.
   a) $f.g$ is simple b) $f + g$ is simple
   c) f^2 is simple d) All of these

8) If C be a cantor set then _____.
 a) $m^*(C) = \infty$ b) $m^*(C) = 1$
 c) $m^*(C) = -1$ d) $m^*(C) = 0$

9) Consider the statements:
 I) Every function of bounded variation is continuous.
 II) If f is a difference of two monotone real valued functions on $[a, b]$ then f is function of bounded Variation on $[a, b]$
 a) Only I is true b) Only II is true
 c) Both I and II are true d) Both I and II are false

10) If $\langle f_n \rangle$ is an increasing sequence of non-negative measurable functions such that ______.
 $\lim_{n\to\infty} f_n = f$ a.e. then
 a) $\int f \geq \lim_{n\to\infty} \int f_n$ b) $\int f \leq \lim_{n\to\infty} \int f_n$
 c) $\int f = \lim_{n\to\infty} \int f_n$ d) $\int f = 1$

B) Write True/False. **06**
1) Fatou's lemma remains valid if 'convergence a.e.' is replaced by convergence in measure.
2) The negative part f^- of a function f is given by $f^-(x) = \max\{f(x), 0\}$
3) Every Borel set is measurable set.
4) Any set with the outer measure different from zero is uncountable.
5) The smallest σ-algebra containing all closed sets and also open intervals is Borel set.
6) If f be a non-negative measurable function on $[a, b]$ such that $\int_a^b f(x)dx = 0$ then $f(x) = 0$ almost everywhere on $[a, b]$.

Q.2 Answer the following. **16**
a) State Egoroff's theorem.
b) If A is singleton set then prove that $m^*(A) = 0$.
c) Show that the given function is measurable.

$$\begin{aligned} f(x) &= x + 4 \quad \text{if } x \geq 2 \\ &= 8 \quad \text{if } x < 2 \end{aligned}$$

d) If $g(x) = f(-x)$ then show that $D^+g(x) = -D\_f(-x)$

Q.3 Answer the following.
a) Prove that the sum, product and difference of two simple function is again simple. **08**
b) If E is a measurable set then prove that the translation $E + y$ is measurable and $m(E + y) = m(E)$. **08**

Q.4 Answer the following.
a) If A be any set and $E_1\ E_2, E_{3,} \dots, E_n$ be a finite sequence of disjoint measurable sets then prove that $m^*(A \cap [U_{k=1}^n E_k]) = \sum_{k=1}^n m^*(A \cap E_k)$ **08**
b) Prove that a function F is an indefinite integral of some integrable function if and only if it is absolutely continuous on $[a, b]$. **08**

Q.5 Answer the following.

a) Show that the outer measure of an interval is its length. **08**

b) If f is a function of bounded variation on $[a, b]$ then prove that, **08**

i) $P_a^b - N_a^b = f(b) - f(a)$

ii) $T_a^b(f) = P_a^b(f) + N_a^b(f)$

Q.6 Answer the following.

a) If f and g are bounded measurable functions defined on a measurable set of finite measure then prove that, **08**

i) $\int_E \alpha f + \beta g = \alpha \int_E f + \beta \int_E g$

ii) $f = g\ a.e \Rightarrow \int_E f = \int_E g$

b) Prove that a function f is of bounded variation on $[a, b]$ if and only if the function f is the difference of two monotone real-valued functions on $[a, b]$. **08**

Q.7 Answer the following.

a) Show that the collection M of all measurable sets forms a σ-algebra. **08**

b) State and prove monotone convergence theorem. **08**

SLR-HO-22

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
MATHEMATICS
General Topology (MSC15203)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Select the correct alternative. **10**

1) In discrete T-space $< X, \Im >, d(\{x\})$ = _____.
   a) $\{x\}$ b) $\emptyset$
   c) X d) $X - \{x\}$

2) Every regular T_1 space is _____.
   a) T_0 b) T_2
   c) T_3 d) None of these

3) Every T_1 space is _____.
   a) T_0 b) T_2
   c) T_3 d) None of these

4) Every compact space is _____.
   a) locally compact b) countably compact
   c) both (a) and (b) d) neither (a) nor (b)

5) Every indiscrete T-space $< X, \Im >$ is _____.
   a) compact
   b) locally compact
   c) countably compact if X is infinite
   d) All of the above

6) Which of the following is not a hereditary property?
   a) Compactness b) Being a Lindelof space
   c) both (a) and (b) d) neither (a) nor (b)

7) If X countable set, then co-countable topology on X resembles with _____.
   a) co-finite topology b) discrete T-space
   c) indiscrete T-space d) none of these

8) In discrete T-space $< X, \Im >$, every subset of X is _____.
   a) an open set but not closed b) a closed set but not open
   c) both open and closed d) Connected

9) Consider the following two statements:
   P: Every compact space is Lindelof
   Q: Every Lindelof space is compact.
   Then
   a) P is true and Q is false b) P is false and Q is true
   c) both P and Q are true d) both P and Q are false

10) A set A in a T-space $< X, \mathfrak{I} >$, is closed iff _____.
   a) $\bar{A} = i(A)$ b) $\bar{A} = A$
   c) $i(A) = A$ d) $i(A) \subset A$

B) State whether true or false. (1 Mark each) **06**

1) If $X = \{a\}$, then discrete topology and indiscrete topology on X are identical.
2) The only connected discrete T-spaces are discrete topology defined on a singleton set X.
3) The usual T-space $< \mathbb{R}, \mathfrak{I}_u >$ is not compact.
4) Every first axiom space is second axiom space.
5) In any co-countable T-space $< X, \mathfrak{I} >$, a subset A of X is open iff $X - A$ is finite.
6) T_2- space is also known as a Hausdorff space.

Q.2 Answer the following. **16**

a) In any T-space $< X, \mathfrak{I} >$, prove that $A \subseteq B \Rightarrow d(A) \subseteq d(B)$, where $A, B \subseteq X$.
b) Prove that closed subset of a compact space is compact.
c) Prove that being a T_0 space is a hereditary property.
d) Define Connected space, Closure of a set, interior of a set, T_2 space.

Q.3 Answer the following. **16**

a) Let X be an uncountable set and define $\mathfrak{I} = \{\emptyset\} \cup \{A \subseteq X | X - A \text{ is countable}\}$. Prove that $< X, \mathfrak{I} >$ is a T-space.
b) If $< X, \mathfrak{I} >, < X^*, \mathfrak{I}^* >$ are two T-spaces and $f: X \to X^*$ is a function, then prove that the function f is continuous on X iff inverse image of every closed set in X^* is closed in X.

Q.4 Answer the following. **16**

a) If $< X, \mathfrak{I} >$ is a T-space and if a connected set C has non-empty intersection with both a set E and complement of E in $< X, \mathfrak{I} >$, then prove that C has a non-empty intersection with boundary of E.
b) Let $< X, \mathfrak{I} >$ be a T-space. Then $< X, \mathfrak{I} >$ is a T_2-space iff intersection of all closed neighborhoods of a point x in X is $\{x\}$.

Q.5 Answer the following. **16**

a) A T-space $< X, \mathfrak{I} >$ is regular iff for any point $x \in X$ and any open set G containing x, there exists an open set H such that $x \in H$ and $\overline{H} \subseteq G$.
b) If $< X, \mathfrak{I} >, < X^*, \mathfrak{I}^* >$ are two T-spaces and $f: X \to X^*$ is a one-one, onto function, then prove that f is a homeomorphism iff $f[c(E)] = c^*[f(E)], E \subseteq X$.

Q.6 Answer the following **16**

a) If $X = \{a, b, c, d\}$, and $\mathfrak{I} = \{\emptyset, \{\mathrm{a, c}\}, \{\mathrm{a, d}\}, \{\mathrm{a}, c, d\}, (a, b), (a, b, c), (a, b, d), X\}$. Then find the derived set of $A = \{b, c, d\}$.
b) Prove that property of being a separable space is a topological property.

Q.7 Answer the following. **16**

a) A T-space $< X, \mathfrak{I} >$, is a T_1 space iff $\{x\}$ is a closed set in X for each $x \in X$.
b) Define locally compact space. Prove that being a locally compact space is a hereditary property.

SLR-HO-23

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April - 2024
MATHEMATICS
Complex Analysis (MSC15206)

Day & Date: Thursday, 16-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The value of $\int_C \frac{(3z+4)}{z(2z+1)}dz,$ where C is the circle $|z| = 1$ is _____.
 a) 3 b) $3\pi i$
 c) $2\pi i$ d) 0

2) Which of the following mappings does not changes the size and shape of the figure?
 a) Rotation, Translation b) Translation, Magnification
 c) Rotation, Magnification d) Inversion, Magnification

3) The function $f(z) = \cos z$ is/has _______.
 a) singularities at $z = \pm\frac{(n+1)\pi}{2}$
 b) singularities at $z = \frac{n\pi}{2}$
 c) analytic only at $z = 0$
 d) analytic everywhere

4) Number of zeros of $f(z) = e^z$ in a finite complex plane is _______.
 a) Zero b) One
 c) Finite d) Countably infinite

5) The residue of the function $f(z) = \frac{\sin z}{z^8}$ at $z = 0$ is ______.
 a) $\frac{1}{7!}$ b) $\frac{-1}{7!}$
 c) 1 d) 0

6) If pole of the bilinear transformation lies on the boundary then the image is ______.
 a) Circle b) Triangle
 c) Straight line d) Parabola

7) If f have an isolated singularity at $z = a$ and $f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n$ is its Laurent expansion about $z = a$ then the residue of f at $z = a$ is _____.
 a) a_{-1} b) a_0
 c) a_{-2} d) a_1

8) Which of the following is an entire function?
 a) $f(z) = \sqrt{x^2+y^2}$ b) $f(z) = x - iy$
 c) $f(z) = z\bar{z}$ d) $f(z) = x + iy$

9) If image of an open set is not open under an analytic function then the function is _____.
   a) not analytic b) constant
   c) non-constant d) not differentiable

10) The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^{-n} z^{2n}$ is _____.
   a) 0 b) 1
   c) ∞ d) $\sqrt{2}$

B) Fill in the blanks. **06**

1) The function $f(z) = \frac{\sin z}{(z-\pi)^2}$ have the pole of order _____ at $z = \pi$.
2) The fixed points of the mapping $f(z) = \frac{z-1}{z+1}$ are _____.
3) A function which has poles as its only singularities in the finite part of the plane is said to be a _____.
4) If $T(z) = \frac{z+2}{z+3}$ then $T^{-1}(z)$ is _____.
5) A polygon with three sides is called ______.
6) If $f: C \to C$ defined by $f(z) = z^2 + 1$ is an analytic function then the set of zeros of the function f is _____.

Q.2 Answer the following **16**

a) Evaluate: $\int_{\gamma} \frac{\cos 2z - e^z}{(z+1)^2 (z+2)^z} dz$ over $\gamma: |z| = 1.5$

b) Define with one example of each.
   i) Singular point of an analytic function
   ii) Zero's of an analytic function

c) If S is a Mobius transformation then prove that S is the composition of Translation, Dilation and Inversion.

d) Find $Res(f;1)$, $Res(f;2)$ for $f(z) = \frac{z^2}{(z-1)^2 (z-2)^2}$

Q.3 Answer the following. **16**

a) If z_1, z_2, z_3, z_4 be the four distinct points in C_∞ then show that the cross ratio (z_1, z_2, z_3, z_4) is real iff all four points lie on a circle or straight line.

b) Find Laurent series expansion of $\frac{z}{(z+1)(z-2)}$ in
   1) $0 < |z+1| < 3$
   2) $1 < |z| < 2$

Q.4 Answer the following.

a) Show that $\int_0^{\pi} \frac{1 + 2\cos\theta}{5 + 4\cos\theta} d\theta = 0$ **08**

b) Let G be an open subset of the complex plane C and $f: G \to C$ be an analytic function. If γ is a closed rectifiable curve in G such that, $\eta(\gamma; w) = 0$; $\forall\, w \in C - G$ then for $a \in G - \{\gamma\}$ prove that, **08**

$$f(a).\eta(\gamma; a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(w)}{w - a} dw$$

`

Q.5 Answer the following.

a) If f is analytic in $B(a, R)$ then show that **10**
$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n; |z-a| < R$
Where, $a_n = \frac{1}{n!} f^n(a)$ and the series has radius of convergence $\geq R$

b) If $\gamma: [0,1] \rightarrow C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that, **06**
$\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$ is an integer.

Q.6 Answer the following.

a) If G be a region and $f: G \rightarrow C$ be an analytic function such that there is a point 'a' in G with $|f(z)| \leq |f(a)| \; \forall \; z \in G$ then show that f is a constant. **06**

b) State and prove Cauchy residue theorem. **10**

Q.7 Answer the following.

a) State and prove Morera's Theorem. **10**

b) Find the Mobius transformation which maps the given points **06**
$z_1 = -1, z_2 = 0$ and $z_3 = 1$ onto the points $w_1 = i, w_2 = 0$ and $w_3 = \infty$.

Seat No.	

Set **P**

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Functional Analysis (MSC15301)

Day & Date: Friday, 10-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) if $T: X \to Y$ be linear transformation then T is continuous iff _____.
   a) T is bounded
   b) T is continuous at origin
   c) T is continuous at any point of X
   d) All of the above

2) If N and N' are normed linear spaces and $T: N \to N'$ then graph of T is given as $T_G =$ _____.
   a) $\{(x, T(x))/x \in N'\}$ b) $\{(x, T(x))/x \in N\}$
   c) $\{(x, T(x))/x \in T\}$ d) $\emptyset$

3) A projection E on a linear space L determines two linear subspaces M and N such that $L =$ _____.
   a) $M + N$ b) $M \cup N$
   c) $M \oplus N$ d) $M \cap N$

4) If d is a metric defined on a vector space X then $d(x + z, y + z)$ $=$ _____ for all $x, y, z, \in X$.
   a) $d(x, y)$ b) $d(x.0)$
   c) $d(y, 0)$ d) $d(x + y, 0)$

5) The norm on $N \times N'$ is defined as $\|(x, y)\| =$_____ for all $x \in N, y \in N'$.
   a) $\|x\| + \|y\|$ b) max $(\|x\|, \|y\|)$
   c) $(\|x\|^p + \|y\|^p)^{\frac{1}{p}}$ d) All of the above

6) It $T: X \to Y$ is a bounded linear transformation then $\|T\|$ is defined as ____.
   a) sup $\{\|T(x)\|/x \in X, \|x\| \leq 1\}$
   b) sup $\{\|T(x)\|/x \in X, \|x\| \geq 1\}$
   c) sup $\{\|T(x)\|/x \in X, \|x\| = 1\}$
   d) both a and c are true

7) Any linear transformation on finite dimensional normed linear space is always _____.
   a) bounded b) discontinuous
   c) continuous d) finite

8) If $\frac{1}{p} + \frac{1}{q} = 1$ then the conjugate space of l_p^n is _____.
   a) l_q^n b) l_p^∞
   c) l_p^n d) l_q^∞

9) By Zorn's lemma, every non-empty partially ordered set in which each chain has an upper bound has a ______.
 a) bound b) supremum
 c) infimum d) maximal element

10) A non-empty subset of a Hilbert space H is said to be an orthonormal set if it contains ______.
 a) orthogonal unit vectors
 b) mutually orthogonal vectors
 c) mutually orthogonal unit vectors
 d) None of these

B) Fill in the blanks. **06**

1) On finite dimensional spaces, all norms are _____.
2) The set of bounded linear transformation $B(X, Y)$ is complete if _____.
3) If X and Y are normed linear spaces, $T: X \to Y$ is an isometry then T preserves _____.
4) A normed linear space X is said to be complete if every Cauchy sequence is _____in X.
5) In a normed linear space, the triangular inequality property is given as, ___.
6) An operator N is said to be normal operator if it commutes with its _____.

Q.2 Answer the following. **16**

a) If V is a normed linear space, d is defined as $d(x, y) = \|x - y\|, \forall x, y \in V$ then prove that $< V, d >$ is a metric space.

b) If $T: X \to Y$ is a linear transformation and T is continuous at any point of X then prove that T is continuous on X.

c) If B and B' are Banach spaces, T is linear transformation of B into B' and T is continuous mapping then prove that its graph T_G is closed.

d) Define orthogonal vectors and orthogonal complement.

Q.3 Answer the following.

a) If H is a Hilbert space and f is an arbitrary functional in H^* then prove that there exists a unique vector $y \in H$ such that $f(x) = < x, y >$ for every $x \in H$ and $\|f\| = \|y\|$. **08**

b) State and prove Riesz Lemma. **08**

Q.4 Answer the following.

a) If $T: X \to Y$ be any linear transformation then prove that T is Continuous on X if and only if T bounded on X. **08**

b) Prove that $B(X, Y)$ is normed linear space where, $\|T\| = sup\{\|T(x)\| : x \in X, \|x\| \leq 1\}$ **08**

Q.5 Answer the following.

a) If X is a normed linear space over the field F and M is closed subspace of X, define $\| . \|_1 : \frac{X}{M} \to R$ by $\| . \|_1 = in f\{\|x + m\| / m \in M\}$ then prove that $\| . \|_1$ is a norm on $\frac{X}{M}$ **08**

b) If P is projection on Banach space B and M and N are its range and null spaces respectively then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$. **08**

Q.6 Answer the following.

a) If x and y are two vectors in a Hilbert space then prove that $4 < x, y >= \|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2$. **08**

b) If X is an inner product space, then prove that $\|x\| =< x, x >^{\frac{1}{2}}$ is a norm on X. **08**

Q.7 Answer the following.

a) If M be a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$. **08**

b) If M is a linear subspace of normed linear space N and f is a linear functional defined on $M, x_0 \notin M$ and M_0 is linear space spanned by M and x_0 then prove that f can be extended to a functional f_0 on M_0 such that $\|f_0\| = \|f\|$ (only for the real scalar field) **08**

SLR-HO-26

Set No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
MATHEMATCIS
Advanced Discrete Mathematics (MSC15302)

Day & Date: Monday, 13-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) A connected subgraph of a graph G which is not properly contained in any other connected subgraph of a graph G is known as _____.
 a) Hamiltonian graph b) Eulerian graph
 c) Planar graph d) maximal connected subgraph

2) $(\{1,2,5,6,10,15,a\},/)$ is a lattice if the smallest value of a is _____.
 a) 30 b) 150
 c) 300 d) 100

3) Which of the following is not a Poset _____?
 a) $(Z,=)$ b) $(Z^+,/)$
 c) $(Z,>)$ d) $(Z,\geq)$

4) The fusion process when applied to adjacent vertices then it _____ the number of connected components of the graph.
 a) increases b) decreases
 c) does not change d) All of these

5) If u and v be vertices of a graph G then which of the following statements is/are true?
 I) Every $u-v$ walk contains a $u-v$ path
 II) Every trail is a path
 a) Only I is true b) only II is true
 c) Both I and II are true d) both I and II are false

6) An element 'a' in the poset P is called a maximal element of P if _____.
 a) $a < x$ for some x in P b) $a < x$ for no x in P
 c) $x < a$ for no x in P d) $x < a$ for some x in P

7) $(n+1).n_{P_r} =$ _____.
 a) $(n+r+1).n_{P_r}$ b) $(n-1).n_{P_r}$
 c) $(n-r+1).n+1_{P_r}$ d) $(n+1).n-1_{P_r}$

8) The connectivity of a connected graph G is one if and only if _____.
 a) $G = K_1$ b) $G = K_2$
 c) G has cut vertex d) both b and c

9) The number of three digits can be formed with the digits $2,3,4,5,6,7$ no digit being repeated are _____.
 a) 60 b) 140
 c) 110 d) 120

10) If G is a connected graph with vertex set V then for each vertex $v \in V$, the eccentricity of vertex v i.e. $e(v)$ is given by ______.
   a) $\max \{ d(u,v)/u \in V \}$ b) $\min \{ d(u,v)/u \in V \}$
   c) $\max \{ d(u,v)/u \in V, u \neq v \}$ d) $\min \{ d(u,v)/u \in V, u \neq v \}$

B) Fill in the blanks. **06**

1) The number of different non-isomorphic spanning trees on the complete graph with 4 vertices are ______.
2) The generating function for the sequence $\{1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \dots\}$ is ______.
3) The characteristic equation of $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$ is ______
4) A simple bipartite graph G, with bipartition $V = V_1 \cup V_2$ in which every vertex in V_1 is joined to every vertex of V_2 is called ______.
5) The number of permutations on 'n' different things taken 'r' at a time, when things may be repeated any number of times is ______.
6) If L and M be lattices then a mapping $f: L \to M$ is called a meet homomorphism if ______.

Q.2 Answer the following. **16**

a) If $(L, \vee, \wedge)$ is a distributive lattice then show that if an element has a complement then this complement is unique.
b) Write a short note on isomorphism of two graphs.
c) Prove that $n_{c_r} + n_{c_{r-1}} = n + 1_{c_r} (0 \leq r \leq n)$
d) In how many ways can 7 boys and 5 girls be seated in a row so that no two girls may seat together.

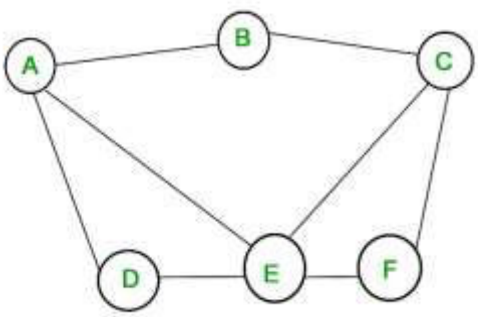
Q.3 Answer the following. **08**

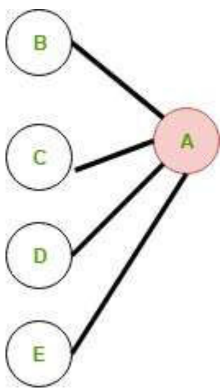
a) Show that a graph G is connected if and only if given any pair u and v of vertices there is path from u to v.
b) Define non-homogeneous recurrence relation and Solve **08**
$y_n - 7y_{n-1} + 12y_{n-2} = n4^n$

Q.4 Answer the following.

a) Find the closed form of generating function of **08**
1) $1, (1+2), (1+2+3), (1+2+3+4), \dots$
2) $1^2, (1^2+2^2), (1^2+2^2+3^2), \dots$

b) Find the distance and diameter of the following graphs. **08**





Q.5 Answer the following.

a) State and prove Bridge Theorem. **10**

b) If $(A, \precsim_1)$ and $(B, \precsim_2)$ are Posets then show that $(A \times B, \precsim)$ is a Poset with partial order defined by. **06**
$(a, b) \precsim (a', b')$ if $a \precsim_1 a'$ in A and $b, \precsim_2 b'$ in B.

Q.6 Answer the following.

a) If G be a graph with n vertices $v_1, v_2, v_3, \dots v_n$ & A denote the adjacency matrix of G with respect to this listing of vertices. Let $B = [b_{i,j}]$ be the matrix $B = A + A^2 + A^3 + \dots + A^{n-1}$. Then show that G is connected graph iff for every pair of distinct indices i, j we have $b_{i,j} \neq 0$. **10**

b) Show that every chain is a distributive lattice. **06**

Q.7 Answer the following.

a) Define finite Boolean algebra and show that D_{42} is a finite Boolean algebra under partial order of Divisibility. **08**

b) Show that a graph G is connected if and only if it has a spanning tree. **08**

SLR-HO-27

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Linear Algebra (MSC15303)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) Which of the following is always true for matrices?
   a) $(AB)^{-1} = B^{-1}A^{-1}$ b) $A^T = A$
   c) $AB = BA$ d) $A * I = I$

2) Which of the following is a subspace of $R3$?
   a) All vectors of the form $(x, 0, 0)$
   b) All vectors of the form $(x, 1, 1)$
   c) All vectors of the form (x, y, z) where $y = x + z + 1$
   d) None of these

3) Which of the following are linear combinations of $u = (0, -2, 2)$ and $v = (1, 3, -1)$?
   a) $(2, 2, 2)$ b) $(0, 0, 0)$
   c) Both a and b d) Neither a nor b

4) Orthonormal set is an orthogonal set with the additional property that, each vector of length _____.
   a) zero b) one
   c) constant d) None of these

5) Every basis of finite dimensional vector space contains _____ number of element.
   a) Same b) Different
   c) Infinite d) None of these

6) Which of the following sets of vectors in $R3$ are linearly independent?
   a) $\{(2,1,2), (8,4,8)\}$
   b) $\{(1,1,0), (1,1,1), (0,1,-1)\}$
   c) $\{(1,3,2), (1,-7,-8), (2,1,-1)\}$
   d) $\{(-2,0,1), (3,2,5), (6,-1,1), (7,0,2)\}$

7) The dimension of zero vector space is _____.
   a) not defined b) 1
   c) 0 d) Infinite

8) If A is a square matrix, then for every eigenvalue of A _____.
   a) The geometric multiplicity is equal to the algebraic multiplicity
   b) The geometric multiplicity is less than or equal to the algebraic multiplicity
   c) The geometric multiplicity is greater than or equal to the algebraic multiplicity
   d) The geometric multiplicity is strictly less than the algebraic multiplicity

9) If A is an n square nilpotent matrix of index k, then its minimal polynomial is _____.
   a) x^{k-1} b) x^k
   c) x^{k+1} d) 0

10) Which of the following is true?
   a) $< u, v + w > = < u, v > + < u, w >$
   b) $< u, v + w > = < v, u > + < w, u >$
   c) Both a and b are true
   d) Neither a nor b is true

B) Write True/False. **06**

1) If the characteristic polynomial of matrix A is $p(\lambda) = (\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3$ then the order of A is 5×6.
2) Any self adjoint operator is normal.
3) If V be the vector space over the field F and subspace $S = \{0\}$ of V then $S^0 = \{0\}$.
4) A form on a vector space V is called bilinear form if V is real vector space.
5) If V be an inner product space over F then for all $x, y \epsilon V$ and $c \epsilon F$ then $\|cx\| = c\|x\|$.
6) If a is orthogonal to b then every scalar multiple of a is orthogonal to b.

Q.2 Answer the following. **16**

a) State and prove Schwarz's inequality.
b) Prove that the similar matrices have the same characteristic polynomial.
c) Prove that every finite dimensional inner product space has an orthonormal basis.
d) Define the following terms:
   i) Annihilating polynomial
   ii) Minimal polynomial
   iii) Normal operator
   iv) Invariant subspace

Q.3 Answer the following.

a) Define linear Transformation and show that the mapping $T: R^3$ onto R^3 defined by $T(a_1, a_2, a_3) = (a_1 - a_2 + 2a_3, 2a_1 + a_2 - a_3, -a_1 - 2a_2)$ is a Linear Transformation. **08**
b) If T be a linear operator on n dimensional vector space V and W be invariant subspace for T then prove that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T and also prove that the minimal polynomial for T_w divides the minimal polynomial for T. **08**

Q.4 Answer the following.

a) If V be finite dimensional vector space over the field F and let W be a subspace of V then prove that $\dim W + \dim W^0 = \dim V$ **10**
b) If T be a linear operator of vector space V, $c \in F$ is characteristic value of T and $f(x)$ is any polynomial over F then prove that $[f(T)](\alpha) = f(c)\alpha$. **06**

Q.5 Answer the following.

a) Find the minimal polynomial of matrix $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$ **08**

b) If T be a linear operator on inner product space V then prove that T is unitary iff the adjoint T^* of T exists and $TT^* = T^*T$. **08**

Q.6 Answer the following.

a) If V and W be finite dimensional inner product spaces over the same field F having the same dimension and T is linear transformation from V into W then prove that the following statements are equivalent: **08**

i) T preserves inner product space
ii) T is an isomorphism
iii) T carries every orthonormal basis for V onto orthonormal basis for W

b) If $\beta_1 = (3,0,4), \beta_2 = (-1,0,7)$ and $\beta_3 = (2,9,11)$ then find the orthogonal and orthonormal basis for R^3 with the standard inner product by using Gram Schmidt orthogonalization process. **08**

Q.7 Answer the following.

a) Prove that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is diagonalizable. **08**

b) Find all possible canonical forms of the matrix A whose characteristic polynomial is given by $(x-2)^3\,(x-5)^2$ **08**

SLR-HO-28

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Differential Geometry (MSC15306)

Day & Date: Friday, 17-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Select the correct alternative. **10**

1) The curve $\alpha(t) = (a\cos t, a\sin t, bt), a > 0, b \neq 0$ represents ______
   a) circle b) ellipse
   c) helix d) ellipsoid

2) A curve $\alpha: I \to E^3$ is called regular if ______
   a) $\alpha(t) = 0, \forall t \in I$ b) $\alpha'(t) = 1$,for some $t \in I$
   c) $\alpha'(t) = 0, \forall t \in I$ d) $\alpha'(t) \neq 0, \forall t \in I$

3) For any nonzero vector v, $\left\| \frac{v}{\|v\|} \right\| =$ ______
   a) 0 b) 1
   c) -1 d) $\|v\|$

4) Osculating plane to a unit speed curve β at the point $\beta(s)$ is spanned by ______
   a) T, N b) T, B
   c) N, B d) T, N, B

5) For the unit speed curve $\alpha: I \to E^3$ with $k > 0$ and torsion $\tau, B' =$
   a) B b) N
   c) kN d) $-\tau N$

6) If curvature for a unit speed curve is identically zero, then the curve is a _____.
   a) circle b) ellipse
   c) straight line d) helix

7) A mapping $\bar{F}: E^3 \to E^3$ preserves distance, then it is known as _____.
   a) symmetry b) constant map
   c) isometry d) None of the above

8) A mapping $T: E^3 \to E^3$ defined by $T(\bar{p}) = \bar{p} + \bar{a}$ is known as ______
   a) translation b) rotation
   c) projection d) orthogonal transformation

9) Which of the following is not a surface?
   a) cone b) closed disc
   c) folded plane d) all of the above

10) For ellipse, its torsion $\tau =$ ______
   a) 1 b) -1
   c) 0 d) None of these

B) State whether True or False. **06**

1) Circle is a regular curve.
2) A surface is called minimal surface if its mean curvature is zero.
3) For sphere of radius *r*, the Gaussian curvature $K = \frac{1}{r}$
4) A curve $\alpha: I \to E^3$ is said to have unit speed if $\|\alpha^n(s)\| = 1$.
5) For a curve α, if $\frac{\tau}{K}$ = constant, then α is a cylindrical helix.
6) If $\alpha :: I \to E^3$ is a regular curve, then $k = \frac{\|\dot{\alpha} \times \ddot{\alpha}\|}{\|\dot{\alpha}\|}$

Q.2 Answer the following. **16**

a) Find the directional derivative $\bar{v}_p[f]$ for $f = x^2yz$ with $p = (1,1,0)$ and $\bar{v} = (1,0,-3)$

b) Find the arc length of the circle $\alpha(t) = (a\cos t, a\sin t . 0)\ 0 \le t\ 2\pi$

c) Define isometry of E^3 and translation.

d) Show that the shape operator of a plane surface is zero.

Q.3 Answer the following.

a) Let f and g be real valued functions on $E^3, \bar{v}_p, \bar{w}_p$ are tangent vectors on E^3 and a, b are real numbers, show that **10**

i) Define directional derivative $\bar{v}_p[f]$.

ii) $(a\bar{v}_p + b\bar{W}_p)[f] = a\bar{v}_p[f] + b\bar{v}_p[f]$

iii) $\bar{v}_p[af + bf] = a\bar{v}_p[f] + b\bar{v}_p[g]$

iv) $\bar{v}_p[fg] = \bar{v}_p[f]g(p) + f(p)\bar{v}_p[g]$

b) Compute $\bar{v}_p[f], \bar{v}_p[g]$ and hence 1- forms for $f = (x^2 - 1)y + (y^2 + 2)z$ and $g = (x^3 - 2)z + (yz - 1)x$. **06**

Q.4 Answer the following.

a) If $V = -yU_1 + xU_3, W = \cos x\, U_1 + \sin x\, U_2$ are the vector fields, then find the covariant derivatives $\nabla_V W,\ \nabla_w V,\ \nabla_v V, \nabla_w W$. **08**

b) If $X: E^2 \to E^3$ is a mapping defined by $X(u,v) = (u+v, u-v, uv)$, show that X is a proper patch and that the image of X is given by $z = \frac{x^2-y^2}{4}$ **08**

Q.5 Answer the following.

a) Find the Frenet apparatus for the curve $\alpha(t) = (1 + t^2, t, t^3)$ at $t = 0$. **08**

b) Find the normal and tangent vector fields on the sphere Σ given by $x^2 + y^2 + z^2 = r^2$ **08**

Q.6 Answer the following.

a) Show that $\bar{F}: E^3 \to E^3$ defined by $\bar{F}(\bar{p}) = -\bar{p}$ is an isometry. Also find its translation and rotation part. **10**

b) Find the unit speed parametrization of a circle $\alpha(t) = (r\cos t, r\text{sin}t, 0)$, $r > 0, 0 \le t \le 2\pi$ of radius r and hence compute the tangent vector field of the curve. **06**

Q.7 Answer the following.

a) Define differential form. If f is a 1-form on $\mathbb{R}^3$, then prove that $f = \sum_i f_i dx_i$, where $f_i = f(\bar{U}_i)$. **08**

b) If $\bar{c}: E^3 \to E^3$ is an orthogonal transformation, then prove that **08**

i) $\bar{c}$ is linear ii) $\bar{c}$ is an isometry

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Measure & Integration (MSC15401)

Day & Date: Thursday, 09-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) If $(X, \mathcal{B}, \mu)$ be a measure space, $E \subseteq X$ then E is called finite measure if ______.
   a) $\mu(X) < \infty$ b) $\mu(E) < \infty$
   c) $\mu(\mathcal{B}) < \infty$ d) All of the above

2) A measure μ on a measurable space X is a saturated measure if _____.
   a) Every subset of X is measurable.
   b) Every locally measurable subset of X is measurable.
   c) Every measurable subset of X is locally measurable
   d) None of the above

3) If f be a non-negative measurable function and $\int f = 0$ then _____.
   a) $f = 0$ almost everywhere b) $f \geq 0$
   c) $f \geq 0$ almost everywhere d) $f = 0$

4) If $\mathcal{A}$ is an algebra then collection of countable union of sets in $\mathcal{A}$ is called as _____.
   a) $\mathcal{A}_\sigma$ b) $\mathcal{A}_\delta$
   c) $\mathcal{A}_\lambda$ d) $\mathcal{A}_{\sigma\delta}$

5) Every signed measure has a ______ Jorden decomposition.
   a) more than one b) infinite
   c) unique d) finite

6) A subset E of X is said to be μ^* measurable, if for any set A _____.
   a) $\mu^*(A) \leq \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   b) $\mu^*(A) \geq \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   c) $\mu^*(A) = \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   d) Both b and c

7) Randon Nikodym theorem holds for _____.
   a) locally measurable sets b) finite measure space
   c) $\sigma -$ finite measure space d) All of these

8) Every signed measure is a _____ of two measures.
   a) sum b) difference
   c) product d) reciprocal

9) A set with positive measure _____.
   a) is a positive set b) not a positive set
   c) need not be a positive set d) negative set

10) Two measures ν_1 and ν_2 on a measurable space are said to be mutually singular if there are disjoint measurable sets A and B such that $X = A \cap B$ and _____.

a) $\nu_1(A) = \nu_2(B) = 0$ b) $\nu_1(B) = \nu_2(A) = 0$
c) $\nu_1(E) = 0$ implies $\nu_2(E) = 0$ d) Both a and b

B) State Ture or False. **06**

1) A set E is said to be positive set with respect to signed measure iff $\nu^{-}(E) = 0$.
2) The condition of σ finitencess is not necessary in Randon Nikodym theorem.
3) Lebesgue outer measure is also μ^* outer measure.
4) A measure on an algebra $\mathcal{A}$ is a measure iff $\mathcal{A}$ is a σ- algebra.
5) The collection $\mathcal{R}$ of measurable rectangles is a σ- algebra.
6) Hahn decompositions of a set is unique.

Q.2 Answer the following. **16**

a) Define measure space and give one example.
b) Show that : Every σ- finite measure is saturated.
c) Prove that: A set E is said to be negative set with respect to signed measure iff $\nu^{+}(E) = 0$.
d) If $\nu_1 << \mu, \nu_2 << \mu$ where ν_1, ν_2, μ are measures then $c_1.\nu_1 + c_2.\nu_2 << \nu$ where c_1, c_2 are constants.

Q.3 Answer the following.

a) Show that the triplet $(R, \mathcal{M}, \mu)$ is a measure space where $\mathcal{M}$ is set of Lebesgue measurable sets and μ is set function defined by $\mu(E) = |E|$ is E is finite, $\mu(E) = \infty$ is E is infinite. **08**
b) State and Prove Monotone convergence theorem. **08**

Q.4 Answer the following.

a) If μ_1 and μ_2 are measures on a measurable space $(X, \mathcal{B})$ such that atleast one of them is finite and $\nu(E) = \mu_1(E) - \mu_2(E)$ for all $E \in \mathcal{B}$ then prove that ν is a signed measure. **08**
b) If ν is a signed measure on measurable space $(X, \mathcal{B})$ then there is a positive set A and negative set B such that $X = A \cup B, A \cap B = \phi$. **08**

Q.5 Answer the following.

a) Prove that: The set of locally measurable sets from σ-algebra. **08**
b) If c is a constant and f, g are measurable function defined on X then prove that $f + c, cf, f + g, f - g$ are measurable functions. **08**

Q.6 Answer the following.

a) Prove that: The class $\mathcal{B}$ of μ^* (outer measure) measurable set is σ- algebra. **08**
b) Prove that: The collection $\mathcal{R}$ of measurable rectangles is a semi algebra. **08**

Q.7 Answer the following.

a) If μ_* is an inner measure and $E \subseteq F$ then prove that $\mu_*(E) \leq \mu_*(F)$. **08**
b) If f and g are non negative extended real valued measurable functions on $(X, \mathcal{B}, \mu)$ and $E \in \mathcal{B}$ then prove that **08**

1) $f \leq g \; a.e \Rightarrow \int_E f d\mu \leq \int_E g d\mu$
2) $\int_E cf d\mu = c\int_E f d\mu$ where $c > 0$.

SLR-HO-31

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April - 2024
MATHEMATICS
Partial Differential Equations (MSC15402)

Day & Date: Saturday, 11-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The problem of finding a harmonic function $u(x, y)$ in D such that it coinsides with f on boundary B is called ______.
   a) Neumann problem b) Wave equation
   c) Dirichlet problem d) Laplace equation

2) A set of those points of a 3-dimensional space which are expressed as function of two parameters is called a ______.
   a) Surface b) Plane
   c) Direction ratio d) Tangent to curve

3) Canonical form of $z_{xx} - 6\, z_{xy} + 9\, z_{yy} + 2p + 3q - z = 0$ is _____.
   a) $\frac{\partial^2 z}{\partial u\, \partial v} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$ b) $\frac{\partial^2 z}{\partial v^2} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$
   c) $\frac{\partial^2 z}{\partial \alpha^2} + \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} + \frac{\partial z}{\partial \beta}$ d) $\frac{\partial^2 z}{\partial \alpha^2} - \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} - \frac{\partial z}{\partial \beta}$

4) The partial differential equation which represents the set of all right circular cones with z-axis as the axis of symmetry is _____.
   a) $yp - xq = 0$ b) $yp + xq = 0$
   c) $xp + yq = 0$ d) $xp - yq = 0$

5) Elimination of a function f from $z = f\left(\frac{y}{x}\right)$ gives a partial differential equation ______.
   a) $x\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ b) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$
   c) $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ d) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

6) Every integral generated by one parameter family of characteristics is an ______.
   a) envelope b) circle
   c) cone d) integral surface

7) The condition that the surfaces $f(x, y, z) = c$ forms a family of equipotential surfaces is that ______.
   a) $\frac{\nabla^2 \mathrm{f}}{|\nabla f|^2} = 0$ b) $\frac{\nabla \mathrm{f}}{|\nabla^2 f|^2} = 0$
   c) $\frac{\nabla^2 \mathrm{f}}{|\nabla f|^2}$ is function of f only d) $\frac{\nabla^2 \mathrm{f}}{|\nabla f|^2}$ is not function of f

8) The equations $f(x,y,p,q)=0$ and $g(x,y,p,q)=0$ are compatible if _____.

a) $\frac{\partial(f,g)}{\partial(x,p)}+\frac{\partial(f,g)}{\partial(y,q)}=0$ b) $\frac{\partial(f,g)}{\partial(x,p)}-\frac{\partial(f,g)}{\partial(y,q)}=0$

c) $\frac{\partial(f,g)}{\partial(y,p)}+\frac{\partial(f,g)}{\partial(x,q)}=0$ d) $\frac{\partial(f,g)}{\partial(y,p)}-\frac{\partial(f,g)}{\partial(x,q)}=0$

9) The integral surface passing through the curve $x_0=0, y_0=s^2, z_0=-s$ of the partial differential equation $(x^2+y^2)p+2xyq=(x+y)z$ is ______.

a) $z^2=y(x+y)^2$ b) $z^2(y-x)=y(x+y)^2$

c) $z(y^2-x^2)=y(x+y)$ d) $z^2(y^2-x^2)=y(x+y)^2$

10) The complete integral of $z^3=pqxy$ is ______.

a) $x^a y^b = exp\left(2\sqrt{\frac{ab}{z}}\right)$ b) $xy = exp\left(\sqrt{\frac{ab}{z}}\right)$

c) $x^a y^b = exp\left(\sqrt{\frac{ab}{z}}\right)$ d) $2x^a y^b = exp\left(\sqrt{\frac{ab}{2z}}\right)$

B) State True or False **06**

1) If $u(x,y)$ is harmonic in a bounded domain D and is continuous on $\overline{D}=D\cup B$, where B is boundary of D. Then $u(x,y)$ attains its minimum on B.
2) The parametric equations of a curve and a surface are unique.
3) The condition $X^{-}.curl\, X^{-}=0$ is equivalent to $P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)-Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial X}\right)=0$
4) The characteristic curves of $4u_{xx}+5u_{xy}+u_{yy}+u_x+u_y=2$ are $y-x=c, 4y-x=d$
5) The Lagrange's auxiliary equation for the partial differential $Pp+Qq=R$ is $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$
6) A function $f(x,y)$ is said to be a homogeneous function of x and y of degree n if $f(\lambda x,\lambda y)=\lambda^n f(x,y)$

Q.2 Answer the following. **16**

a) Prove that a necessary and sufficient condition that there exists a relation between two functions $u(x,y)$ and $v=v(x,y)$ a relation $F(u,v)=0$ or $u=H(v)$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)}=0$.

b) Prove that the solution of Dirichlet problem if it exists is unique.

c) Show that there always exists an integrating factor for a Pfaffian differential equation in two variables.

d) Find a partial differential equation by eliminating arbitrary constant from $z=x+ax^2y^2+b$

Q.3 Answer the following.

a) Find the complete integral of $pxy+pq+qy-yz=0$ by Charpit's method. **08**

b) Obtain D-Alembert's solution of the one-dimensional wave equation which describes the vibration of an infinite string. **08**

Q.4 Answer the following.

a) Show that $(x-a)^2+(y-b)^2+z^2=1$ is a complete integral of $z^2(1+p^2+q^2)=1$ then by taking $b=2a$ show that the subfamily is $(y-2x)^2+5z^2=5$ which is a particular solution. Show further that $z=\pm 1$ are the singular integrals. **08**

b) Find the condition that a one parameter family of surfaces forms a family of equipotential surfaces. **08**

Q.5 Answer the following.

a) Prove that a necessary and sufficient condition that the Pfaffian differential equation $\bar{X}\,\overline{dr}=0$ be integrable is that $\bar{X}\,curl\,\bar{X}=0$. **10**

b) Solve $xu_x+yu_y={u_z}^2$ by Jacobi's method. **06**

Q.6 Answer the following.

a) Find the complete integral of $(p^2+q^2)x=pz$ and hence find the integral surface through the curve $x=0, z^2=4y$. **10**

b) Show that the equations $f=p^2+q^2-1=0$ & $g=(p^2+q^2)x-pz=0$ are compatible and find the one parameter family of common solution. **06**

Q.7 Answer the following.

a) Find the general solution of $2x(y+z^2)p+y(2y+z^2)q=z^3$ **08**

b) Reduce the equation $u_{xx}+x^2u_{yy}=0$ to a canonical form. **08**

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Integral Equations (MSC15403)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Select the correct alternative: **10**

1) Which of the following is not a separable kernel?
a) $K(x,t) = \sin h\,(x+t)$ b) $K(x,t) = \cos h\,(x+t)$
c) $K(x,t) = e^{xt}$ d) All of the above

2) An integral equation $g(x)y(x) = f(x) + \lambda \int_a^b K\,(x,t)\,y(t)dt$ is said to be of the first kind if ______.
a) $g(x) = 0$ b) $g(x) = 1$
c) $f(x) = 0$ d) $f(x) = 1$

3) Solution of $y(x) = x - \frac{x^2}{2}\int_0^x y\,(t)dt$ is ______.
a) $y(x) = 1$ b) $y(x) = 0$
c) $y(x) = x$ d) $y(x) = -x$

4) A Fredholm integral equation $\cos x = y(x) + \lambda \int_0^1 xty(t)dt$ is _____.
a) homogeneous second kind
b) non-homogeneous second kind
c) homogeneous first kind
d) non-homogeneous first kind

5) Which of the following is a convolution type kernel?
a) $K(x,t) = (t-x)^2$
b) $K(x,t) = e^{(t-x)}$
c) $K(x,t) = \sin(t-x) + (t-x)$
d) All of the above

6) Which of the following type of integral equation don't have eigenvalues?
a) Non-homogeneous Fredholm integral equation
b) Non-homogeneous Volterra integral equation
c) Homogeneous Volterra integral equation
d) all of the above

7) Which of the following kernel is symmetric?
a) $K(x,t) = i(xt)$ b) $K(x,t) = i(x-t)$
c) $K(x,t) = i(x+t)$ d) $K(x,t) = e^{ixt}$

8) The solution of $y(x) = 1 - x^2 + \int_0^x xy\,(t)dt$ is ______.
a) 0 b) 1
c) x d) $1 + x$

9) An nth iterated kernel of a Fredholm integral equation $y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$ is ________.

a) $K_n(x,t) = \int_a^b K(x,z)K_{n-1}(z,t)dz$

b) $K_n(x,t) = \int_a^b K(x,z)K_{n-2}(z,t)dz$

c) $K_n(x,t) = \int_t^x K(x,z)K_{n-1}(z,t)dz$

d) $K_n(x,t) = \int_a^x K(x,z)K_{n-2}(z,t)dz$

10) Eigen values of symmetric kernel of a homogeneous Fredholm integral equation are _____.

a) always imaginary
b) always positive
c) always negative
d) always real

B) State whether True or False. **06**

1) $\int_0^x y(t)dt^n = \int_0^x \frac{(x-t)^{(n-1)}}{(n-1)} y(t)dt$

2) Initial value problem gets converted into Fredholm integral equation.

3) Volterra Integral equations with convolution type kernel are solved by Laplace transform.

4) If $K(x,t) = x; a = 0, b = 2$ is a kernel of a Fredholm integral equation, then the second iterated kernel $K_2(x,t) = 2x$

5) Every homogeneous Fredholm integral equation always have a solution.

6) Every boundary value problem possesses a Green function.

Q.2 Answer the following. (4 Marks each) **16**

a) Define first kind, second kind, third kind and homogeneous second kind Volterra integral equation.

b) Show that: $y(x) = 3$ is solution of $\int_0^x (x-t)^2 y(t)dt = x^3$

c) Find eigenvalue and eigen function $y(x) = \lambda \int_0^1 e^x e^t y(t)dt$.

d) Convert the following IVP into an integral equation
$y'' + y = 0, y(0) = y'(0) = 0$

Q.3 Answer the following.

a) Convert the IVP $y''(x) - 3y'(x) + 2y(x) = 4\sin x, y(0) = 1, y'(0) = -2$ **08**

b) Solve by using resolvent kernel method: $y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xty(t)dt$. **08**

Q.4 Answer the following. **16**

a) Solve by using resolvent kernel method: $y(x) = (1+x^2) + \int_0^x \frac{1+x^2}{1+t^2} y(t)dt$.

b) Convert the following IVP into integral equation using substitution method: $y'' + xy' + y = 0; y(0) = 1, y'(0) = 0$.

Q.5 Answer the following.

a) Find the Green's function for the BVP $y'' = 0; y(0) = y(l) = 0$. **10**

b) Solve using Laplace transform: $Y(t) = e^{-t} - 2\int_0^t \cos(t-x)Y(x)dx$. **06**

Q.6 Answer the following.

a) Convert the following into an integral equation: $y'' + xy = 1; y(0) = y(1) = 0$ **08**

b) Solve: $y(x) = \cos x + \lambda \int_0^\pi \sin(x-t)y(t)dt$. **08**

Q.7 Answer the followings. **16**

a) Find the eigenvalues and eigen functions for $y(x) = \lambda \int_0^{2\pi} \sin(x+t)y(t)dt$. **10**

b) Solve by the method of successive approximations: **06**
$y(x) = 1 + \int_0^x (x-t)y(t)dt; y_0(x) = 1$.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Operations Research (MSC15404)

Day & Date: Thursday, 16-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The non-negative variable which is substracted from the left hand side of the constraints to convert it into equations is called ______ variable.
   a) slack b) surplus
   c) artificial d) dummy

2) Consider the following statements:
   I) The closed ball in R^3 is a convex set.
   II) A hyperplane in R^n is a convex set
   a) only I is true b) only II is true
   c) both are true d) both are false

3) A saddle point in game exists when-
   a) maximin value=maximax value
   b) minimax value= minimum value
   c) minimax value=maximin value
   d) all of the above

4) The best use of linear programming problem is to find an optimal use of ______.
   a) money b) machine
   c) manpower d) all of the above

5) The dual of the primal problem is obtained by, ______
   a) transposing the co-efficient matrix and reverting the inequalities
   b) interchanging the role of constant terms and the co-efficients of the objective function
   c) minimizing the objective function instead of maximizing it
   d) all of the above

6) The dual simplex method works towards ______ while simplex method works towards _____.
   a) optimality, feasibility
   b) feasibility, optimality
   c) boundedness, basic solution
   d) finiteness, basic solution

7) Simplex method is developed by American mathematician_____.
   a) Frank Wolf b) Martin Beale
   c) Ralph E. Gomory d) George Dantzig

8) The region bounded by $x_1 + x_2 \le 0$ is _____.
   a) Line   b) Circle
   c) Line segment   d) Unbounded region

9) Games which involve more than two players are called ______.
   a) conflicting games   b) negotiable games
   c) n-person game   d) all of the above

10) The set of all feasible solution of a linear programming problem is ______ set.
   a) Convex   b) Concave
   c) Strictly convex   d) Strictly concave

B) Fill in the blanks. **06**

1) A game is said to be fair if both upper and lower values of the game are same and are _____.
2) If a primal LPP has a finite solution then the dual LPP should have ______ solution.
3) To convert ≥ inequality constraints into equality constraints, we must add a ______.
4) A quadratic form $Q(X)$ is positive definite iff $Q(X)$ is _____ for all $x \neq 0$
5) For a maximization problem, the objective function coefficient in Big M method for an artificial variable is _____.
6) Simplex method is an iterative method to solve _____ programming problem.

Q.2 Answer the following **16**

a) Prove that: A hyperplane in R^n is a convex set.
b) Prove that: The dual of the dual of a given primal is primal.
c) Write general form of Quadratic programming problem.
d) Define :
   i) Extreme point of convex set
   ii) Convex hull

Q.3 Answer the following.

a) Solve the following problem by Simplex method. **08**
$\text{Max } Z = 5x_1 + 3x_2$ subject to the constraints $3x_1 + 5x_2 \le 15$ $5x_1 + 2x_2 \le 10$ and $x_1, x_2 \ge 0$

b) Write an algorithm of Gomory's cutting plane method. **08**

Q.4 Answer the following.

a) If X is any feasible solution to the primal problem and W is any feasible solution to the dual problem then prove that $CX \le b^T W$. **08**

b) Prove that: The collection of all feasible solutions to linear programming problem constitutes a convex set whose extreme point corresponds to the basic feasible solution. **08**

Q.5 Answer the following.

a) Solve the following problem by Dual Simplex method. **10**

Min $Z = 2x_1 + x_2$ subject to the constraints $3x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3\ \ and\ \ x_1, x_2 \geq 0$

b) Find the saddle point and solve the game: **06**

	Player B B_1	B_2	B_3	B_4
Player A A_1	1	7	3	4
A_2	5	6	4	5
A_2	7	2	0	3

Q.6 Answer the following.

a) Solve by using Wolfe's Method. **08**

Max. $Z_x = 4x_1 + 6x_2 - 2x_1^2 - 2\,x_1x_2 - 2x_2^2$ subject to $x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$

b) Solve the following problem. **08**

Max $Z = -2x_1 - x_2$ subject to the constraints $3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,$ $x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$

Q.7 Answer the following.

a) Write Beale's Algorithm for solving Quadratic Programming problem. **08**

b) Prove that: The intersection of two convex sets is a convex set. **08**

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Seat No.	

Set P

M.Sc. (Semester-IV) (New) (CBCS) Examination: March/April - 2024
MATHEMATICS
Numerical Analysis (MSC15408)

Day & Date: Saturday, 18-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The digits that are used to express a number is called ______.
   a) significant digit b) significant figure
   c) both a and b d) error

2) How many real roots does the equation $\sin x - x = 0$ have?
   a) 2 b) 3
   c) 1 d) infinite

3) The eigenvalues of 4×4 matrix $[A]$ are given as $2, -3, 13,$ and 7 then the $|\det(A)|$ is ______.
   a) 546 b) 25
   c) 19 d) 37

4) The method of false position is also known as _____.
   a) Secant Method b) Newton-Raphson Method
   c) LU-decomposition d) Regula Falsi Method

5) The equation $f(x)$ is given as $x^3 + 4x + 1 = 0$. Considering the initial approximation at $x = 1$. Then the value of x_1 is given as ______.
   a) 0.6712 b) 0.1856
   c) 0.1429 d) 1.8523

6) The root of the equation $f(x) = 0$ lies in interval (a, b) if ______.
   a) $f(a) > 0, f(b) = 0$ b) $f(a) > 0, f(b) > 0$
   c) $f(a) < 0, f(b) < 0$ d) $f(a) > 0, f(b) < 0$

7) Gauss-Seidal iterative method is used to solve ______.
   a) differential equation
   b) system of linear equations
   c) system of non-linear equations
   d) partial differential equation

8) If E_R is an relative error then the percentage error is given by ______.
   a) $E_p = E_R \times 100$ b) $E_p = -E_R \times 100$
   c) $E_p = E_R \times 10$ d) $E_p = \frac{E_R}{100}$

9) The convergence of which of the following method is depends on initial assumed values?
   a) False position b) Newton-Raphson Method
   c) Gauss Seidel method d) Euler's method

10) For decreasing the number of iterations in Newton Raphson method ____.
   a) The value of $f'(x)$ must be increased
   b) The value of $f''(x)$ must be decreased
   c) The value of $f'(x)$ must be decreased
   d) The value of $f''(x)$ must be increased

B) Write true/false. **06**
1) If A is invertible matrix then determinant of A is zero.
2) LU decomposition is more efficient than Gauss elimination when solving for the inverse of a matrix.
3) In Gauss elimination method, upper triangular matrix of coefficient matrix can be found by row transformation.
4) The order of convergence of the Bisection method is 2.
5) The Newton Raphson method fails if $f'(x)$ is zero.
6) The root/roots of the equation $e^x - 4x = 0$. lying between 0 and 1.

Q.2 Answer the following **16**
a) Describe rate of convergence of Newton Raphson method.
b) Round of the number 86.5250 to four significant figures and compute percentage and relative error.
c) Define eigen values and eigen vectors.
d) Find the largest eigen value of $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ by using Rayleigh's power method.

Q.3 Answer the following.
a) Solve the following system of equations. **08**
$3x + 2y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8$
by using LU decomposition method.
b) Find the root of the equation $x^4 - x - 10 = 0$ by Newton-Raphson method. **08**

Q.4 Answer the following.
a) Explain the construction of Gauss Seidal method. **08**
b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by Secant method. **08**

Q.5 Answer the following.
a) Obtain the solution upto 5^{th} approximation of the equation $\frac{dy}{dx} = x + y$ such that $y = 1$ when $x = 0$ and find $y(1)$ by using Picard's method. **10**
b) Find all the eigen values of the matrix $\begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$. **06**

Q.6 Answer the following.
a) Solve the following system of equations. **10**
$x + y + z = 2, x + 2y + 3z = 5, 2x + 3y + 4z = 11$
by using Gauss elimination method.
b) Write a note on Euler's modified method. **06**

Q.7 Answer the following.
a) Reduce the matrix $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ to the tridiagonal form. **08**
b) Explain the second order Runge-Kutta method. **08**

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Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
MATHEMATICS
Probability Theory (MSC15410)

Day & Date: Saturday, 18-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) If F_1 and F_2 are two fields, then _____ is always a field.
   a) $F_1 \cap F_2$ b) $F_1 \cup F_2$
   c) both (a) and (b) d) neither (a) nor (b)

2) If $\{A_n\}$ is decreasing sequence of sets, then it converges to ____.
   a) $\liminf A_n$ b) $\limsup A_n$
   c) both (a) and (b) d) None of the above

3) If P is probability measure defined on (Ω, A), then $P(\varphi) =$ _____ (φ is empty set)
   a) Zero b) One
   c) 0.5 d) 0.3325

4) The $\sigma -$ field generated by the intervals of the type $(-\infty, x), x \in R$ is called _____.
   a) Standard $\sigma -$ field b) Borel $\sigma -$ field
   c) Closed $\sigma -$ field d) None of these

5) If $x \epsilon A$ implies $x \epsilon B$, then _____.
   a) $A\ C\ B$ b) $B\ C\ A$
   c) $A = B$ d) All of these

6) Which of the following is the weakest mode of convergence?
   a) convergence in r^{th} mean
   b) convergence in probability
   c) convergence in distribution
   d) convergence in almost sure

7) Monotonic sequence of sets _____.
   a) Always converges
   b) Converges, only if it is bounded above
   c) Converges, only if it is bounded below
   d) Converges, only if it is bounded

8) If μ is measure defined on (Ω, A) such that $\mu(\Omega) = k(k\ is\ finit)$, then μ is called _____ measure.
   a) Good measure b) Finite measure
   c) Total finite measure d) Signed measure

9) Probability measure is continuous from ______.
   a) Above b) Below
   c) Both (a) and (b) d) Either above or below

10) If for events A and B, $A \cup B = \Omega$, then these events are called as _____.
   a) exhaustive b) Exclusive
   c) both (a) and (b) d) Complementary

B) Fill in the blanks. **06**

1) The collection of all subsets of Ω is called as _____.
2) If $\{A_n\}$ is a sequence of independent events, such that $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P\left(\underline{lim} A_n\right) =$ _____.
3) Lebesgue measure of a singleton set $\{k\}$ is _____.
4) The sequence of sets $\{A_n\}$, where $A_n = (0, 2 + \frac{1}{n})$ converges to _____.
5) A random variable X is integrable, if and only if _____ is integrable.
6) If for two independent events A and B, $P(A) = 0.2$, $P(B) = 0.4$, then $P(AUB) =$ _____.

Q.2 Answer the following **16**

a) Write a short note on Lebesgue measure.
b) Discuss Convergence in distribution.
c) State
   i) Liapouniv's CLT
   ii) Lindeberg-Feller CLT
d) Define characteristic function. Show that it is real iff X is symmetric about origin.

Q.3 Answer the following.

a) Define probability measure. Prove that if P and Q are probability measures then **08**
$P^*(A) = \alpha\, P(A) + (1 - \alpha)\, Q(A), 0 \leq \alpha \leq 1$ is a probability measure.
b) Define a field. Examine for the class of finite or co-finite sets to be a field. **08**

Q.4 Answer the following.

a) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence. **08**
b) Define a measurable function. Examine for indicator function of a set to be measurable. **08**

Q.5 Answer the following.

a) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P\,(A_n) < \infty$ $Show\ that$ **08**

$P\left(\overline{lim}\, A_n\right) = 0.$

b) Define almost sure convergence and convergence in r^{th} mean. If $X_n \xrightarrow{r} X$ then prove that $X_n \xrightarrow{p} X$ **08**

Q.6 Answer the following.

a) Define mapping. Let X be a mapping defined on sample space Ω. Let A and $B \subset \Omega$ such that $A \cap B = \phi$. Prove or disprove: $X(A) \cap X(B) = \phi$. **08**
b) State and prove monotone convergence theorem. **08**

Q.7 Answer the following.

a) Find lim inf and lim sup of following sequence of sets. **08**

1) $A_n = \left(1 + \frac{1}{n}, 3 + \frac{2}{n}\right)$

2) $A_n = (0, a + b(-1)^n), a > b > 0$

b) Define a measurable function. Suppose $X(\omega)$ takes three different values **08**

such that $X(\omega) = \begin{cases} C1, \omega \epsilon A_1 \\ C2, \omega \epsilon A_2 \\ C3, \omega \epsilon A_3 \end{cases}$

and $C_i \epsilon IR, i = 1,2,3$. Discuss measurability of X.