# Solapur University, Solapur.

## **Ph.D.** Course Work in Mathematics

## (w.e.f. June 2014)

# Paper II: Recent Trends In Mathematics.

## UNIT 1: GRAPH THEORY.

Matrix representation of graphs, Incidence matrix and properties, Circuit matrix and its properties, Application to a switching network, Cutset matrix and it's properties, Adjacency matrix and it's properties, Walk's in graphs and diagraphs.

### **Reference:**

1. Deo Narsingh : Graph Theory (Prentice Hall of India Pvt. Ltd. (2000)

### UNIT 2: FUZZY MATHEMATICS

Fuzzy sets Vs Crisp sets- Basic types, concepts, properties, Representations and Extension principle of Fuzzy sets; Operations on Fuzzy sets-Complements, intersections, unions and its combinations and aggregation; Fuzzy Relations- Binary, Equivalence, Compatibility, Ordering and Fuzzy morphisms; Fuzzy Logic- Classical logic, Multivalued logic, Fuzzy propositions, Fuzzy Quantifiers.

### **Reference:**

1.	Klir G.J., Yuan Bo	:	Fuzzy sets and Fuzzy logic
			(Prentice Hall of India)
2.	Ross Timothy	:	Fuzzy Logic
			(Wiley India)

### UNIT 3: LATTICE THEORY

Posets, Equivalence of two definitions of Lattice, Properties of Lattice, Homomorphisms, Ideals and Dual ideals, Distributive Lattice, Browerian Lattice, Modular Lattice, Stone Lattice and Pseudo-Complemented Lattice.

### **Reference:**

1.	Gratzer:	Distributive Lattices-Basic Concepts (Freeman and Co.)
2.	Birkhoff:	Lattice Theory Amer.Math.Soc.Collg.Pub.no.25

### UNIT 4: ALGEBRA OF MODULES

Theory of modules-Definitions, Examples, Submodules, Direct sums, R-homomorphisms, Quotient modules, Completely reducible module.

### **Reference:**

1. Bhattacharya, Jain, Nagpaul: Basic Abstract Algebra (Cambridge University Press)

## UNIT 5: ORDINARY DIFFERENTIAL EQUATIONS

Existence of solutions, Uniqueness, The method of differential equations, The nth order equation, Dependence of solutions on initial conditions and parameters, Complex Systems, Extension of the idea of solution, Max and Min of solutions.

### **Reference:**

1. Coddingtun E.A., Levinson N.: Theory of Ordinary Differential Equation

### UNIT 6: ANALYSIS

Continuity of functions on  $\mathbb{R}^2$ , Differentiability of  $F : \mathbb{R}^m \to \mathbb{R}^n$ , Properties of Differential, Directional derivatives, Continuously differentiable functions, Inverse function theorem, Implicit function theorem.

### **Reference:**

1. "Mathematical Analysis" by T.M. Apostol

# Paper III: Advanced Topics in Mathematics

### **Unit 1**: Number Theory

Fermat's factorization method, the little theorem, Fibonacci numbers and properties, Certain identities involving fibonacci numbers.

### **Reference:**

1. Burton David M. : Elementary Number Theory

## Unit 2: Tensor Techniques

Summation Convention, Transformations, Covariant, Contra variant and Mixed tensors, Cartesian tensors, Metric tensors, Algebra of tensors, Christoffel symbols, Derivatives of metric tensors, Laws of transformation of Christoffel symbols.

### **Reference:**

**1.**Paria G. : A Textbook of Matrix and Tensors (Scholar's Publications, Indore.)

### **Unit 3 :** Functional Analysis

Finite Dimensional Spectral Theory, Matrices, Determinants and spectrum of an operator, The spectral Theorem, Fixed Point Theorem and application to analysis.

### **Reference:**

1. Simmons G.F. : Introduction to Topology and modern Analysis (Tata Mc Graw-Hill Publications)

### **Unit 4 :** Algebra of Rings

Noetherian and Artinian modules and rings,  $Hom(\oplus M_i, \oplus M_i)$ , Wedderburn-Artin Theorem, Uniform modules, Primary modules and Noether-Lasker Theorem, Decomposition Theorem, Uniqueness of the Decomposition.

### **Reference:**

1. Bhattacharya, Jain, Nagpal: Basic Abstract Algebra (Cambridge University Press)

### **Unit 5 :** Partial Differential Equations

Second order elliptic equations, Elliptic Equations, Weak solutions, Existence of weak solutions, Regularity, Maximum principles, Eigen values and Eigen functions.

### **Reference:**

1.Evans L.C. : Partial Differential Equations (Graduate Studies in Mathematics Vol. 17,AMS 1998)

## Unit 6 : Algebraic Topology.

The fundamental Groups, Homotopic paths, Contractible and simply connected spaces, The covering Homotopy property for  $S^1$ , The degree of a loop, Equivalence of loops, Isomorphism between the Fundamental Groups and the group Z of Integers.

### **Reference :**

1. Croom Fred H. : Basic concepts of Algebraic Topology (Springer Verlag publications)

## Ph.D. (Course Work) Nature of Question Paper Pattern

- Ph.D. कोर्सवर्कसाठी फक्त Long Answer व Short Answer असेच प्रश्न असतील.
- Ph.D. (Course work) प्रश्नपत्रिकेत कोणताही External Option व Objective प्रश्न असणार नाहीत.
- एकूण प्रश्न ५ x गुण २० = १०० गुण
- प्रश्न क्रमांक १ ते ५
- (A) दिर्घोत्तरी प्रश्न (१० गुण)
  - (B) Answer Any two out of three (प्रत्येकी ५ गुण)

या प्रश्नपत्रिकेच्या स्वरुपामुळे Internal Option हा २५% राहतो.