SLR-MA – 507

Seat No.	

M.Sc. (Part – I) (Semester – I) (New CBCS) Examination, 2016
MATHEMATICS (Paper – I)
Object Oriented Programming Using C++

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternatives : **10**

1) Where does the execution of the program starts ?
A) Main function B) User-defined function
C) Void function D) None of these

2) Which is more effective while calling the functions ?
A) Call by value B) Call by pointer
C) Call by reference D) None of these

3) What is the scope of the variable declared in the user defined function ?
A) Whole program B) Only inside the { } block
C) Both A) and B) D) None of these

4) Which keyword is used to declare the friend function ?
A) Firend B) Myfriend C) Classfriend D) Friend

5) What is default visibility mode for members of classes in C++ ?
A) Private B) Public C) Protected D) None of these

6) Which of the following operator is overloaded for object cout ?
A) >> B) << C) + D) =

7) A template can be considered as a kind of
A) Micro B) Macro C) Maxcro D) None of these

8) Where can the default parameter be placed by the user ?
A) Leftmost B) Rightmost C) Both A) and B) D) None of these

9) _______ are run time anomalies or unusual conditions that a program may encounter while executing.
A) Template B) Virtual Function
C) Exception D) None of these

10) ________ operator cannot overload.
A) + B) – C) << D) ?:

P.T.O.

B) State whether following statement is **true** or **false** : **4**

1) You can use C++ as a procedural, as well as an object-oriented, language.
2) Constructors are invoked automatically when the objects are created.
3) A class can be derived from another derived class is called multiple inheritance.
4) A virtual function can be friend of another class.

2. A) Attempt the following questions : **8**

i) What are the advantages of OOPs ?
ii) What do you mean by user defined data types ? Explain with example.

B) Write a short note on following : **6**

i) Inline function
ii) Scope resolution operator

3. Attempt the following questions. **14**

A) What is arrays of objects ? Explain with suitable example.

B) What is constructor ? Explain parameterized constructor with example.

4. Attempt the following questions : **14**

A) What is function overloading ? Explain with suitable example.

B) Write a C++ program to overload unary minus operator.

5. Attempt the following questions. **14**

A) What is virtual function ? Explain the rules for virtual functions.

B) Write a program to implement a sphere class with appropriate members and member function to find the surface area and the volume.

(Surface = $4 \pi r^2$ and volume = $\frac{4}{3} \pi r^3$)

6. Attempt the following questions. **14**

A) What is template ? Explain function template.

B) Write a C++ program to demonstrate single inheritance.

7. Attempt the following questions. **14**

A) What is manipulator ? Explain width( ), precision( ) and fill( ) manipulators with example.

B) What is friend function ? Explain it with example.

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SLR-MA – 508

Seat No.	

M.Sc. – I (Semester – I) Examination, 2016
MATHEMATICS (New) (CBCS)
Algebra – I (Paper – II)

Day and Date : Thursday, 31-3-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**one** mark **each**) :

1) In a ring of integers associate of 5 are __________

2) If o(G) = 77 then G is __________

3) A group G is Nilpotent if __________

4) $< x^2 - 3 >$ is a __________ ideal in Q[x].

B) State **true** or **false** (**one** mark **each**) :

1) If F is a field, the units in F[x] are precisely the non zero elements of F.

2) Every UFD is PID.

3) The normalizer in G of a subgroup H of G is always a normal subgroup of G.

4) No group order 21 is simple.

C) Define the following (**two** marks **each**) :

i) Solvable group

ii) Unique factorization domain

iii) Conjugate class.

P.T.O.

2. a) Resolve $x^2 + 1$ into factors over the field Z_5. **(3+4+3+4)**

   b) Prove that every nilpotent group is solvable.

   c) Find units in following rings.

   1) Z_5 2) $Z \times Z$

   d) Let F be a field. If an $a \in F$ is a zero of $f(x) \in F[x]$. Then prove that $(x – a)$ is a factor of $f(x)$ in $F[x]$.

3. a) Check whether Group of order 45 is simple or not. **(7+7)**

   b) Prove that every ring R is an R-module over itself.

4. a) If $\cup$ is collection of all units in a ring $< R, +, .>$ with unity then prove that $(\cup, .)$ is a group. **(7+7)**

   b) Prove that every finite group G has at least one composition series.

5. a) Derive the class equation of group G. **(7+7)**

   b) Let G be a group and G' be the derived subgroup of G then prove that,

   1) G' is normal in G

   2) G/G' is abelian

   3) If N is any normal subgroup of G then G/N is abelian iff $G' \subseteq N$.

6. a) A group G is solvable iff there exits some positive integer k such that $G^{(k)} = \{e\}$. **(6+8)**

   b) Prove that Every PID is UFD.

7. a) The polynomial $x^4 + 4$ can be factored into linear factors in $Z_5[x]$. Find this factorization. **(7+7)**

   b) State and prove third sylow theorem.

SLR-MA – 509

Seat No.	

M.Sc. – I (Semester – I) (New) (CBCS) Examination, 2016
MATHEMATICS (Paper – III)
Real Analysis – I

Day and Date : Saturday, 2-4-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**one** mark **each**) :

   1) The partial derivatives describes the rate of change of a function in the direction of each __________

   2) If f is a real valued function then the Jacobian matrix consists of ___________

   3) The infimum of set of all upper sums is called _____________

   4) A bounded function f having ___________ number of discontinuities on [a, b] is integrable.

   B) State **true** or **false** (**one** mark **each**) :

   1) Every integrable function is continuous.

   2) Every closed interval is connected.

   3) Existence of all directional derivatives at a point imply continuity at a point.

   4) The oscillatory sum is always positive.

P.T.O.

C) Define following terms (**two** marks **each**) :

i) Riemann sum.

ii) Open mapping.

iii) Directional derivative.

2\. a) Determine whether $f(x) = \frac{1}{x+1}$ is Riemann integrable on [0, 1] and justify your answer. **3**

b) Find the directional derivative of f(x, y, z) = $x^2z + y^2z^3 - xyz$ in the direction of v = – i + 3k. **3**

c) Define Riemann sum and integrability of a function by Riemann sum. **4**

d) If f is differentiable at c then prove that f is continuous at c. **4**

3\. a) If P^* is a refinement of P then prove that **7**

i) $L(P^*, f) \geq L(P, f)$.

ii) $U(P^*, f) \leq U(P, f)$.

b) State and prove second mean value theorem. **7**

4\. a) State and prove implicit function theorem. **10**

b) Let A be an open subset of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives D_jf_i on A. If f is one to one on A and if $J_f(x) \neq 0$ for each x in A then prove that f(A) is open. **4**

5\. a) If f_1 and f_2 are two bounded and integrable functions on [a, b] then prove that their product $f_1.f_2$ is also bounded and integrable. **7**

b) If a function f is bounded and integrable on [a, b] then prove that the function F defined as $F(x) \int_a^x f(t)\,dt$; $a \leq x \leq b$ is continuous on [a, b]. Further more if f is continuous at a point c of [a, b] then prove that F is derivable at c and $F'(c) = f(c)$. **7**

6. a) Let S be an open subset of R^n and let $f : S \rightarrow R$ be a real valued function with finite partial derivatives D_1f, D_2f ..... D_nf on S. If f has a local maximum or a local minimum at a point c in S then prove that $D_kf(c) = 0$ for each k. **7**

   b) Solve : $\int_2^3 x^3\, dx$. **7**

7. a) Prove that the function f defined on [0, 1] as

   $f(x) = 2n$, if $x = \frac{1}{n}$ where n = 1,2,....

   = 0, otherwise

   is not Riemann Integrable. **7**

   b) Find directional derivative of following functions :

   i) $f(x, y, z) = \log(1 + x^2 + y^2 + z^2)$ at the point P (1, – 1, 1) in the direction v(2, – 2,3).

   ii) $f(x, y) = x^2 - 2y^2$ at the point P (– 2, 3) in the direction v (– 1, 2). **7**

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Seat No.	

M.Sc. – I (Semester – I) (New CBCS) Examination, 2016
MATHEMATICS (Paper – IV)
Differential Equation

Day and Date : Tuesday, 5-4-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Questions No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**) : **5**

1) If ϕ_1, ϕ_2 are two solutions of L(y) = 0 then $W(\phi_1, \phi_2)$ (x) is

a) $\phi_1 \phi_2' - \phi_1' \phi_2$ b) $e^{-a_1(x-x_0)}$

c) (a) and (b) d) None of the above

2) The indicial polynomial of Bessel equation of order α is

a) $r^2 - \alpha^2$ b) $r^2 + \alpha^2$ c) $r^2 + r - \alpha^2$ d) None of these

3) If r_1, r_2 are roots of characteristic polynomial of L(y) = 0 then solution of L(y) = 0 is

a) $\phi_1(x) = e^{r_1 x}$ b) $\phi_2(x) = e^{r_2 x}$

c) $\phi(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ d) (a), (b) and (c)

4) Three linearly independent solutions of $y''' - 3y' + 2y = 0$ are

a) e^x, e^x, e^{-2x} b) e^x, xe^x, e^{-x} c) e^x, xe^x, e^{-2x} d) e^x, xe^x, e^{2x}

5) Let α, β are given constants and x_0 is some real no.

L(y) = $y'' + a_1 y' + a_2 y = 0, y(x_0) = \alpha, y'(x_0) = \beta$ is

a) Initial value problem b) Boundary value problem

c) Cauchy's problem d) Neuman problem

B) Fill in the blanks (**one** mark **each**) : **7**

1) Two solutions of $3y'' + 2y' = 0$ are $\phi_1(x)$ = ________ and $\phi_2(x)$ = _______.

2) The singular point of $3x^2y'' + x^6y'' + 2xy = 0$ is _______ It is singular point.

3) The solution of $2x^2y'' + xy' - y = 0$ for x > 0 is ___________.

4) The Bessel equation of order α is __________.

5) In general particular solution ψ_p of second order non-homogenious equation (differential) with constant coefficient is ___________.

6) If r is a root of multiplicity m of a characteristic polynomial p(r) of n^{th} order LDE with constant coefficients then p(r) = p′ (r) = ___________ $p^{(m-1)}(r)$ = _______.

7) The Lipschitz condition is _________.

C) Write **True** or **False** : **2**

1) If $\phi_1, \ldots, \phi_n$ are linearly dependent functions on an interval I then any subset of them forms a linearly dependent set of functions on I.

2) The functions 1, x, x^3 are linearly dependent.

2. a) Suppose $\alpha \pm i\beta$ is a complex root of the characteristic polynomial of $y'' + a_1y' + a_2y = 0$ where the constants a_1, a_2 are real, α, β are real. Show that $\alpha = -a_1/2, \beta^2 = a_2 - \left(a_1^2/4\right)$. **3**

b) Prove that for any real x_0 and constants α, β there exists a solution ϕ of L(y) = $y'' + a_1y' + a_2y = 0, y(x_0) = \alpha, y'(x_0) = \beta$ on $-\infty < x < \infty$. **4**

c) Find all real valued solution of $y'' + y = 0$. **3**

d) Find second independent solution ϕ_2 of $y'' - \frac{2}{x^2}y = 0$ $(0 < x < \infty)$ if first solution is $\phi_1(x) = x^2$. **4**

3. a) Let x_0 be a point in I. Prove that two solutions ϕ_1, ϕ_2 of L(y) = 0 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$. **7**

   b) Find all solutions of $y^{(4)} + 16y = 0$. **7**

4. a) Suppose f is a real valued function defined on rectangle or strip S such that $\frac{\partial f}{\partial y}$ exists, is continuous on S and $\left|\frac{\partial f}{\partial y}(x, y)\right| \leqq k$, ((x, y) in S) for some k > 0. Prove that f satisfies a Lipschitz condition on S with Lipschitz constant k. **7**

   b) Find the solution ϕ satisfying $y''' - 4y' = 0.\ \phi(0) = 0, \phi'(0) = 1, \phi''(0) = 0$. **7**

5. a) Prove that there exists at most one solution ϕ of L(y) = 0 satisfying

   $\phi(x_0) = \alpha_1,\ \phi'(x_0) = \alpha_2, \ldots, \phi^{(n-1)}(x_0) = \alpha_n$ where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are constants and x_0 be any number. **7**

   b) Solve $y' = xy, y(0) = 1$ by method of successive approximations. **7**

6. a) Let $\phi_1, \phi_2, \ldots, \phi_n$ be n solutions of L(y) = 0 on an interval I and let x_0 be any point in I. Prove that

   $$W(\phi_1, \phi_2, \ldots, \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(+)dt\right] W(\phi_1, \phi_2, \ldots, \phi_n)(x_0).$$ **7**

   b) Show that $f(x, y) = 4x^2 + y^2$ satisfies Lipschitz condition on the set $S : |x| \leq 1, |y| < \infty$. **7**

7. a) Derive Bessel function of zero order of the first kind. **7**

   b) Compute the solution of $y^{(3)} + y^{(2)} + y^{(1)} + y = 1$. **7**

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SLR-MA – 511

Seat No.	

M.Sc. – I (Semester – I) Examination, 2016
MATHEMATICS (New) (CBCS)
Classical Mechanics (Paper – V)

Day and Date : Thursday, 7-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q.* ***1*** *& Q.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q.* ***3*** *to Q.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. Fill in the blanks (**one** mark **each**). **14**

1) Number of generalized co-ordinates to describe simple pendulum is/are ________________.

2) Rigid body motion is an example of _________ constraints.

3) The problem in calculus of variations getting minimum time is called ______________.

4) In the definition of Lagrangian which is independent of generalized velocity ____________.

5) An expression which represent constraints with inequality is called as ___________.

6) If q is cyclic co-ordinate in Lagrangian L then $\frac{\partial L}{\partial q}$ = ______________.

7) Out of 10 generalized co-ordinates in L, 4 are cyclic then number of Routh's equations are ______________.

8) If we solve Lagrange's equation of motion for L then number of arbitrary constants in L is/are ________________.

9) If a point mass is moving freely in the plane then its degrees of freedom are ______________.

10) If $\bar{r}_i = \bar{r}_i(q_1, q_2, ..., q_n, t)$ then $\sum_{j=1}^{k} \frac{\partial \bar{r}_i}{\partial q_j} . \dot{q}_j + \frac{\partial \bar{r}_i}{\partial t}$ is ________________.

11) $\sum_{j=1}^{n} p_j \dot{q}_j - L$ is called ________________.

12) If $L = m(\dot{x}^2 + \dot{y}^2) - k(x^2 + y^2)$ then $P_x =$ ______________.

13) Generalized force $Q_j =$ ________________.

14) If $H = \frac{1}{2} ml^2 (P_\theta)^2 + mgl \cos\theta$ then $\dot{\theta} =$ ______________.

2. a) State Hamilton's principle. **(3+3+4+4)**

b) Find expression of generalized momentum.

c) Define Eulerian angles.

d) Find Euler's equation for $\int_a^b F(y, y')\, dx$.

3. a) Derive Hamilton's canonical equations of motion from a variational principle. **10**

b) Derive the Newton's laws of motion from Lagranges equation. **4**

4. a) Find the extremal of the functional $I = \int_{t_1}^{t_2} \frac{1}{2}(x\dot{y} - y\dot{x})\, dt$, subject to the constraint $J = \int_{t_1}^{t_2} (\dot{x}^2 + \dot{y}^2)^{1/2}\, dt = l$. **7**

b) Derive Euler's equation for the extremum of the functional $\int_{x_1}^{x_2} F(x, y, y')\, dx$. **7**

5. a) Derive Euler's equations for the motion of a rigid body with one point fixed. **9**

   b) Write a short note on Caley-Klein parameters. **5**

6. a) Find Lagrange's equations of motion of a double pendulum. **8**

   b) Find the equation of motion of one-dimensional harmonic oscillator in Hamiltonian formulation. **6**

7. a) Derive Lagrange's equations of motion for the system of n particles using D'Alembert's principle. **9**

   b) Find the extremal of the functional : **5**

$$I = \int_{x_1}^{x_2} \frac{1+y^2}{(y')^2}\, dx \, .$$

SLR-MA – 512

Seat No.	

M.Sc. (Part – I) (Semester – I) (Old CGPA) Examination, 2016
MATHEMATICS (Paper – I)
Object Oriented Programming using C++

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: i) Q.* ***1*** *and Q.* ***2*** *are* ***compulsory.***
ii) Attempt ***any three*** *questions from Q.* ***3*** *to Q.* ***7.***
iii) Figures to ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative: **10**

1) Strings are
 a) Bunch of character b) Number of words
 c) Numbers d) None of these

2) Which data type is used to represent real numbers ?
 a) Integer b) Number
 c) Float d) None of these

3) Which of the following is a library function ?
 a) xyz () b) Pqr(a,b) c) Studinfo() d) clrscr()

4) Which loop is called as Entry Control Loop ?
 a) While b) Do while
 c) For d) None of these

5) Message passing is done with the help of
 a) Object b) Class c) Structure d) Union

P.T.O.

6) Do while loop is called as
   a) Entry control loop   b) Exit control loop
   c) Unconditional   d) None of these

7) Constructors should be declared in
   a) Public section   b) Private section
   c) Protected section   d) None of these

8) Ability to take more than one form is called as
   a) Polymorphism   b) Virtual functions
   c) Inheritance   d) Abstract class

9) The symbol of Unary operator is
   a) '?:'   b) '++'
   c) '+'   d) None of these

10) The operator which needs two operands is called as
   a) Binary operator   b) Ternary operator
   c) Unary operator   d) Boolean operator

B) State whether following statements are **True** or **False** : **4**

1) Private data members are directly accessed through object.

2) The function which calls itself called as recursion.

3) The value which is changing during execution of program is called as constant.

4) Operator overloading is the example of compile time polymorphism.

2. A) Write a short note on following : **8**

i) Abstract Class

ii) Static Data Member.

B) Answer the following : **6**

i) How does Ternary Operator Work ?

ii) Comment on 'Dynamic Binding'.

3. Answer the following :

   A) What is data type ? Explain the different data types used in C++. **7**

   B) Create a Class Employee having name, Department, Salary as data members and empinfo as member function. Write a code for empinfo which accepts input for data members. **7**

4. Answer the following :

   A) Describe the concepts of polymorphism with example. **7**

   B) Differentiate between Public and Private Visibility modes. **7**

5. Answer the following :

   A) Explain in detail the concept of data encapsulation. **7**

   B) How does virtual function work ? Explain. **7**

6. Answer the following :

   A) Discuss the concept of virtual base class. **7**

   B) Write a C++ Program to demonstrate parameterized constructors. **7**

7. Answer the following :

   A) Write a note on 'Friend Function'. **7**

   B) Write a C++ Program to demonstrate concept of call by reference. **7**

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SLR-MA – 513

Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2016
MATHEMATICS
Algebra – I (Paper – II)

Day and Date : Thursday, 31-3-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

*N.B. : 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to **7**.*
*3) Figures to **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) The group of order 77 is ____________

2) The group of order 15 has exactly ___________ sylow 5-subgroups.

3) F is field iff F[x] is ______________

4) Let $f(x) = a_0 + a_1x + ... + anx^n$ be a polynomial in F[x] an element t in F is a zero of f(x) if ____________

5) If O(G) = n then class equation of G is O(G) = ______________

B) State whether following statements are **true** or **false** (**one** mark **each**) :

1) Derived subgroup of Finite Group is normal.

2) Any non-constant polynomial $f(x) \in F(x)$ of degree n has exactly n zeros in F.

3) Subgroup of solvable group need not to be solvable.

4) If H and K are any two subgroup of a finite group G then $O(HK) = O(H) \cdot O(K)$.

5) In finite group G any sylow P- subgroup of group G is unique iff it is normal subgroup of group G.

C) Choose the correct alternatives (**one** mark **each**) :

1) Consider the two statements :

I) 1 and –1 are units in Z.

II) Every Non-zero element in a field F is a unit in F. Then always

a) Only (I) is true b) Only (II) is true

c) Both (I) and (II) are true d) Both (I) and (II) are false

2) Let Z be the ring of integers then, Z is

a) Ring with unity b) Field

c) Integral domain d) Commutative ring with unity

3) Let A and B be submodules of an R-module M then consider the two statements,

I) $A \cap B$ is submodule of M

II) A + B is submodule of M then,

a) Only (I) is true b) Only (II) is true

c) Both (I) and (II) are true d) Both (I) and (II) are false

4) The polynomial $P(x) = 1 + x + x^2 + x^3 + x^4$ over Z is,

a) Reducible over Q b) Irreducible over Q

c) Reducible over Z d) Irreducible over $\mathbb{R}$

2. a) If $N \Delta G$, then show that the derived subgroup of N is also a normal subgroup of G. **4**

b) Show that $f(x) = x^3 + 3x + 2$ is irreducible over Q. **4**

c) State Zassenhaus lemma. **3**

d) Define the term Euclidean domain with suitable example. **3**

3. a) State and prove First Sylow Theorem. **8**

b) Check whether the group of order 36 is simple or not. **6**

4. a) State and prove the division algorithm for the two polynomial's of unequal degree. Defined on field F. **8**

   b) Let H be a subgroup of G and $N \Delta G$ then show that $\frac{HN}{N} \cong \frac{H}{H \cap N}$. **6**

5. a) Define an ideal of ring, prove that an ideal $< p(x) > \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F. **7**

   b) Let $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$

   $g(x) = x^2 + 2x - 3$ in Z[x]

   find q(x) and r(x)
   such that
   $f(x) = q(x) \cdot g(x) + r(x)$ and deg $r(x) < 2$. **7**

6. a) Show that any two subnormal series of a group have isomorphic refinements. **7**

   b) Let R be commutative ring with unity show that the relation in R defined by "a is an associate of b" is an equivalence relation. **7**

7. a) Show that Every Euclidean Domain is a P.I.D. **7**

   b) Prove that any homomorphic image of R module M is isomorphic with its suitable quotient module. **7**

SLR-MA – 514

Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2016
MATHEMATICS
Real Analysis – I (Paper No. – III)

Day and Date : Saturday, 2-4-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*

2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*

3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) The Riemann-Stieltjes integral reduces to Riemann integral if ____________.

2) The existence of all partial derivatives at a point fails to imply ____________ at that point.

3) A bounded function f is integrable on [a,b], if the set of its points of discontinuity has only finite number of _____________.

4) A function $f : S \to T$ is called open mapping if for every open set A in S, the image f (A) is _____________.

5) A non-vanishing $J_f(a)$ guarantees that f is _____________ on a neighbourhood of a.

6) The total derivative of linear function is ______________.

7) The statement $\int_a^b f\,dx$ exists implies that function f is ____________ and ____________ over [a, b]. **7**

B) State **true** or **false** (**each** have **one** mark) :

1) If a function f is monotonic on [a, b] then it is integrable on [a, b].

2) Continuity is necessary condition for integrability of function.

P.T.O.

3) If f is continuous and positive on [a, b] then $\int_a^b f\,dx$ is also positive.

4) The mean value theorem functions from R' to R' state that $f(y) - f(x) = f'(z).(y - x)$ where $z \in (x, y)$ is false in general for vector valued functions from R^n to R^m (m>1).

5) A function whose second derivative is positive will be concave up.

6) Let f be a function with domain D. Then f has an absolute mini value on D at point 'd' if $f(d) \geq f(x)\ \forall x \leftarrow D$.

7) If f is a bounded on [a, b] then to every $\varepsilon > 0$ there corresponds $\delta > 0$ such that $L(P, f) < \underline{\int_a^b} f\,dx - \varepsilon$. **7**

2. a) Show that x^2 is integrable on any interval [0, K]. **4**

b) For any bounded function f show that $\underline{\int} fdx \leq \overline{\int} fdx$. **3**

c) $F(x, y) = \frac{xy}{x+y}$; $x \neq 0, y \neq 0$

$= 0$; $x = 0\ y = 0$

Verify whether directional derivative exist at 'O' in the direction of $u = (u_1, u_2)$. **4**

d) Let $f : R^2 \to R^3$ be defined by f(x, y) = (sinx. cosy, sinx siny, cosx.cosy) Determine Df (x, y). **3**

3. a) Prove that if f is bounded function on [a, b] then to every $\varepsilon > 0$ there corresponds $\delta > 0$ such that

i) $U(p, f) < \overline{\int_a^b} f\,dx + \varepsilon$

ii) $L(p, f) > \underline{\int_a} f\,dx - \varepsilon$

For every partition p of [a, b] with norm $\mu(p) < \delta$. **7**

b) If f is integrable on [a, b] then prove that f^2 is also integrable on [a, b]. **7**

4. a) Show that $\int_1^2 f\,dx = \frac{11}{2}$ where $f(x) = 3x + 1$. **7**

b) Prove that every continuous function is integrable. **7**

5. a) If a function f is continuous on [a, b] then prove that there exists a number ξ in [a, b] such that $\int_a^b f\,dx\ f(\xi)\ (b-a)$. **7**

b) Let S be an open connected subset of R^n and let $f : S \to R^m$ be differentiable at each point of S. If $f'(c) = 0$ for each c in S then prove that f is constant on S. **7**

6. a) If P* is a refinment of P then prove that $\cup(P^*, f, \alpha) \leq \cup(P, f, \alpha)$. **7**

b) Prove that a function f is integrable with respect to α on [a, b] if and only if for every $\varepsilon > 0\ \exists$ a partition P of [a, b] such that $\cup(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$. **7**

7. a) Let $f : S \to R^m$ be a differentiable at an interior point c of S where $S \subseteq R^n$. If $V = v_1u_1 + v_2u_2 + \ldots + v_nu_n$ where $u_1, u_2, \ldots, u_n$ are the unit co-ordinate vectors in R^n then prove that $f'(c)(v) = \sum_{k=1}^{n} v_k\ D_k f(c)$. **7**

b) State and prove the necessary and sufficient condition for the integrability of a bounded function f is that to every $\varepsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of [a, b] with norm $\mu(P) < \delta$,

$U(P, f) - L(P,f) < \varepsilon$. **7**

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Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2016
MATHEMATICS
Paper – IV : Differential Equations

Day and Date : Tuesday, 5-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to question No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose correct alternative (**one** mark **each**) : **4**

i) If r_1 is a root of multiplicity m of characteristic polynomial p(r) of n^{th} order LDE with constant coefficients then $p(r_1) = 0$ $p^1(r_1) = 0$ ......$p^{(m-1)}(r_1) =$ ___________

a) 1 b) 0
c) n d) n!

ii) $x^2y'' + xy' + (x^2 - n^2)y = 0$ is

a) Legendre equation
b) Euler equation
c) Bessel equation
d) Laplace equation

iii) The solution of the equation $(1-x^2)y'' - 2xy' + n(n-1)\,y = 0$ have convergent power series expansion on ___________

a) |x| < 0
b) |x| > 0
c) |x| < 1
d) |x| > 1

iv) The regular singular points of $x(x-1)^2(x+2)y'' + x^2y' - (x^3 + 2x - 1) = 0$ are

a) 0, 1 b) 0, 2
c) 0, –2, d) 1, –2

P.T.O.

B) Fill in the blanks (**one** mark **each**) : **5**

i) The function g is analytic at x_0, if g can be expanded in power series about x_o which has _____________ radius of convergence.

ii) The LipSchitz condition for f(x, y) on set S is _____________

iii) The indicial polynomial of Euler's equation of order two is given by ________

iv) The solutions of $y'' - 2y' + 4y = 0$ are $\phi_1(x)$ ________ and $\phi_2(x)$ = ________

v) A polynomial solution P_n of degree n of $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ satisfying _____________ is called Legendre polynomial.

C) State **true** or **false** (**one** mark **each**) : **5**

i) x = 1 is regular singular point of Eulers differential equation.

ii) L is a differential operator which operates on functions which have n derivatives on I and transforms such a function ϕ into a function $L(\phi)$.

iii) The functions cosx and sinx are linearly dependent.

iv) The value of $\frac{d}{dx}\left[X^n J_n(x)\right] = X^n J_{n-1}(X)$.

v) Generating function of Legendre's polynomial is $(1 - 2Xh + h^2)^{-½}$.

2. a) Give the geometrical interpretation of $\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$. **3**

b) Find the Wronskian of $y''' - 3r_1y'' + 3r_1y' - r_1^3y = 0$. **3**

c) Show that the successive approximations ϕ_k, defined by integral equation exist as continuous functions on

$I : |x - x_0| \leqq \alpha$ = minimum {a, b/M} and $(X, \phi_R(x))$ is in R for x in I. **4**

d) Prove that the Legendre polynomial is given by $Pn(x) = \frac{1}{2^n . n!} \frac{d^n}{dx^n}(x^2 - 1)^n$. **4**

3. a) State and prove existence theorem for second order equation. **7**

b) Show that all the successive approximations for the problem $y' = y^2$, y(0) = 1 exist for all real x. Also find the solution of $y' = y^2$, y(0) = 1. **7**

4\. a) Prove that two solutions ϕ_1, ϕ_2 of L(y) = 0 are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I. **7**

b) Find all solutions of $x^2y'' + xy' - 4\pi y = x$ for |x| > 0. **7**

5\. a) Prove that $W(\phi_1.....\phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)dt\right] W(\phi_1, \phi_2,...\phi_n)(x_0)$ if $\phi_1,...\phi_n$ be n solutions of L(y) = 0 on an interval I and let x_0 be any point in I. **7**

b) Find solution of $4y'' - y = e^x$. **7**

6\. a) If ϕ_1 is a solution of $y'' + a_1(x)y' + a_2(x)y = 0$ and $\phi_1(x) \neq 0$ on I then prove that a second solution is given by $\phi_2(x) = \phi_1(x)\int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp\left[-\int_{x_0}^{s} a_1(t)dt\right] ds$. **7**

b) Solve $y'' + 2iy' + y = x$. **7**

7\. a) Derive Bessel function of zero order of the first kind. **7**

b) Find that solution ϕ satisfying $y'' + (4i+1)y' + y = 0, y(0) = 0, y'(0) = 0$. **7**

SLR-MA – 516

Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2016
MATHEMATICS
Classical Mechanics (Paper – V)

Day and Date : Thursday, 7-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

N.B. *: 1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any 3** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. Fill in the blanks (**one** mark **each**) : **14**

1) A curve encloses maximum area is a _________
2) The relation between Δ and δ is _________
3) If the internal and external forces acting on the system of a particle are conservative, then _________ of the system is conserved.
4) Constraints which are not explicitly dependent on time is called _________
5) Euler-Lagrange's equations are useful to find _________
6) For a conservative system, when the constraints are Rheonomic, Hamiltonian represents _________ but does not represent _________
7) A rigid body is defined as _________
8) If N particles are moving in a space with holonomic constraints expressed in k equations then the no. of independent co-ordinates are _________
9) Routh's procedure are used to solve the problems involving _________
10) If the kinetic energy of a system of a particle is $T = \sum_j \sum_k a_{jk} \dot{q}_j \dot{q}_k$ then $2T =$ _________
11) The Hamilton's canonical equations of motion are given by _________
12) If the total force F is zero then _________ is conserved.
13) The finite rotations of a rigid body about the fixed point of a body are _________
14) The force is said to be conservative if _________

P.T.O.

2. a) Show that a co-ordinate which is cyclic in the Lagrangian is also cyclic in the Hamiltonian.

   b) Write short note on infinitesimal rotations.

   c) Explain generalized force.

   d) Show that gravitational force is conservative. **(4+3+3+4)**

3. a) Obtain Lagrange's equation of motion for simple pendulum.

   b) State and prove principle of least action. **(7+7)**

4. a) Obtain the Euler's equation of motion of a rigid body when one point of the rigid body remains fixed.

   b) State and prove basic lemma in calculus of variation. **(7+7)**

5. a) Obtain a matrix which specifies the orientation of a rigid body in terms of Cayley-Klein parameters.

   b) On which curve the functional

   $$I\left[y(x)\right] = \int_0^{\pi} \left[y^{12} - y^2 + 4y\cos x\right].\,dx$$

   with $y(0) = 0,\ y(\pi) = 0$ be extremized ? **(7+7)**

6. a) Use Hamilton's procedure to find the equation of motion of a particle moving under the inverse square law of attractive force.

   b) If for a cyclic generalized co-ordinate dq_j represents the total rotation of the system of a particle along some axis $\hat{n}$ then, show that, total angular momentum of a system is conserved along $\hat{n}$. **(7+7)**

7. a) Obtain the Lagrangian for double pendulum.

   b) Obtain the equation of motion of a particle moving under the influence of a central force. **(7+7)**

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SLR-MA – 517

Seat No.	

M.Sc. – I (Semester – II) (New – CBCS) Examination, 2016
MATHEMATICS
Algebra – II (Paper No. – VI)

Day and Date : Wednesday, 30-3-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**one** mark **each**).

1) Characteristic of a field is either _________ or _______ .

2) Relation between $Q(\sqrt{2})$ and $Q(3+\sqrt{2})$ is ____________ .

3) The splitting field of $x^2 - 4$ over Q is ____________ .

4) If K is a normal extension of F then o(G(K, F)) = _________ .

5) The factor group $\frac{Z_6}{<3>}$ is of order __________ .

6) The element $\sqrt{1+\sqrt{3}}$ is algebraic over Q of degree _________ .

7) $\left[Q\left(\sqrt{2}, \sqrt[3]{5}\right): Q\right] =$ ___________ .

B) State **true** or **false** (**one** mark **each**) :

1) Angle of 10^0 is constructible.

2) Every finite field is perfect.

P.T.O.

3) Every group G is isomorphic to itself.

4) It is not possible to have a homomorphism of some infinite group into some finite group.

5) $\sqrt{\pi}$ is algebraic over R of degree 2.

6) One can find any constructible in a finite number of steps by starting with a given segment of unit length and using a straight edge and a compass.

7) For fields $F \subseteq E \subseteq K$, $G(K, E) \leq G(K, F)$.

2. a) Find the order of $G\left(Q\left(2^{\frac{1}{3}}, i\sqrt{3}\right), Q\left(2^{\frac{1}{3}}\right)\right)$.

   b) Show that $\alpha = \sqrt{1+\sqrt[3]{2}}$ is algebraic over Q and find its degree.

   c) Prove that : The set of all constructible real numbers forms a subfield of field of real numbers.

   d) Let K be an extension of Q. Show that any automorphism of K must leave every element of Q fixed. **(3+3+4+4)**

3. a) Show that the polynomial $x^2 + 1$ is irreducible in $Z_3[x]$.

   b) Let K be a normal extension of a field F of characteristic zero and T is be a subfield of K containing F then prove that T is a normal extension of F iff

   $\sigma(T) \subseteq T \ \forall \sigma \in G(K, F)$. **(7+7)**

4. a) Prove that : Every finite extension K of a field F is algebraic and may be obtained from F by the adjunction of finitely many algebraic elements.

   b) Find the Galois group of $x^3 - 2$ over the field of rational numbers. **(7+7)**

5. a) Determine the splitting field of $x^4 - 2$ over the field of rational numbers. What is the degree of this spliting field ?

   b) If K is a field and if $\sigma_1, \sigma_2, \ldots, \sigma_n$ are distinct automorphisms of K then prove that it is impossible to find the elements $a_1, a_2, \ldots, a_n$ not all zero in K such that $a_1 \sigma_1(u) + a_2\sigma_2(u) + \ldots + a_n\sigma_n(u) = 0 \; \forall u \in K$ **(6+8)**

6. a) Prove that : The polynomial $f(x) \in F[x]$ has a multiple root iff f(x) & $f'(x)$ have a non-trivial common factor.

   b) If a is constructible number then prove that $\sqrt{|a|}$ is constructible. **(7+7)**

7. a) Prove that : The Galois group of polynomial over a field is isomorphic to a group of permutations of its roots.

   b) Show that the polynomial $x^3 - 8$ is solvable by radicals over Q. **(7+7)**

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SLR-MA – 518

Seat No.	

M.Sc. I (Semester – II) (New CBCS) Examination, 2016
MATHEMATICS
Paper – VII : Real Analysis – II

Day and Date : Friday, 1-4-2016 Max.Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *i) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*

ii) Attempt ***any three*** *questions from Q. no.* ***3*** *to Q. no.* ***7****.*

iii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Fill in the blanks : **7**

i) If $m^*(A) = 0$ then $m^*(A \cup B) =$ ____________

ii) If A and B are any two sets then $\chi_{A \cap B} =$ ____________

iii) If f is absolutely continuous function on [a, b] and if $f'(x) = 0$ a.e., then the function f is ____________

iv) If $D^+f(x) = D_+f(x)$ then their common value is denoted by ____________

v) If $T_a^b(f)$ is total variation of f over [a, b] and if $T_a^b(f) < \infty$ then the function f is of ____________

vi) If φ is a simple function defined on any measurable set E then

$\int_E \varphi =$ ____________

vii) If f is a real - valued function defined on [a, b], and if $a = x_0 < x_1 < ... < x_k = b$ is any subdivision of [a, b] then p = ____________

b) State whether following is **True** and **False** : **7**

i) The set [0, 1] is countable.

ii) If f and g are bounded measurable functions defined on a set E of finite measure. If $f \le g$ a.e., then $\int_E f \le \int_E g$.

P.T.O.

iii) The lower left-hand derivative of the function f at x is given by

$$D^{-} f(x) = \lim_{h \to 0^{+}} \frac{f(x) - f(x-h)}{h}.$$

iv) Every absolutely continuous function is the indefinite integral of its derivative.

v) Fatou's Lemma remains invalid if "convergence a.e." is replaced by "convergence in measure".

vi) If $f_n(x) \to f(x)$ for each $x \in E$ then the sequence $\langle f_n \rangle$ converges pointwise to f on E.

vii) A function φ defined on interval (a, b) is convex if for each $x, y \in (a, b)$ and for each $0 \leq \lambda \leq 1$ such that $\varphi(\lambda x + (1-\lambda)y) \geq \lambda\varphi(x) + (1-\lambda)\,\varphi\,(y)$.

2. a) If A is countable set then prove that $m^*(A) = 0$. **3**

   b) Show that $D^+(-f(x)) = -D_+f(x)$. **4**

   c) If f and g are two measurable real-valued functions defined on the same domain then prove that f + g is also measurable. **4**

   d) State Egoroff's theorem. **3**

3. a) If $\{A_n\}$ is a countable collection of sets of real numbers then prove that

   $$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^*(A_n).$$ **7**

   b) If f is a non-negative function which is integrable over a set E, then prove that for given $\varepsilon > 0$ there is a $\delta > 0$ such that for every $A \subset E$ with $m(A) < \delta$ we

. have $\int_A f < \varepsilon$. **7**

4. a) If E_1 and E_2 are measurable, then show that

   $$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$$ **7**

   b) Prove that a function F is an indefinite integral if and only if it is absolutely continuous. **7**

5. a) If φ is a continuous function on (a, b) and if $D^{+}\varphi$ is nondecreasing, then prove that φ is convex function. **7**

b) If f is bounded and measurable function on [a, b] and if $F(x) = \int_a^x f(t)\,dt + F(a)$, then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$. **7**

6. a) If f is of bounded variation on [a, b] then prove that $T_a^b(f) = P_a^b(f) + N_a^b(f)$. **7**

b) If $\langle E_n \rangle$ is an infinite decreasing sequence of measurable sets and if $m(E_1)$ is finite then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$. **7**

7. a) Prove that a function f is of bounded variation on [a, b] if and only if the function f is the difference of two monotone real-valued functions on [a, b]. **7**

b) If a function f is integrable on [a, b] and if $\int_a^x f(t)\,dt = 0$ for all $x \in [a, b]$, then prove that f(t) = 0 a.e., in [a, b]. **7**

SLR-MA – 519

Seat No.	

M.Sc. (Part – I) (Semester – II) (New) (CBCS) Examination, 2016
MATHEMATICS (Paper – VIII)
General Topology

Day and Date : Monday, 4-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : 1) *Attempt **any five** questions.*
2) *Q. No. **1** and Q. No. **2** are **compulsory**.*
3) *Attempt **any three** from Q. No. **3** to Q. No. **7**.*
4) *Figure to the **right** indicates **full** marks.*

1. A) Fill in the blanks. **7**

i) A subset of topological space is said to be closen if _________

ii) For any set A in topological space $\langle X, \tau \rangle$ $\overline{A}$ = _________

iii) A topological space $\langle X, \tau \rangle$ is called separable iff _________

iv) A finite topological space $\langle X, \tau \rangle$, is a T_1 space if and only if _________

v) A metric space is separable if and only if _________

vi) Every continuous mapping of a compact space into Hausdorff space is _________

vii) Closed subspace of Lindelof space is _________

B) State whether the statements are **true/false** : **7**

i) Any subset of indiscrete topological space $\langle X, \tau \rangle$ is compact.

ii) Any compact subset of R is closed in $\langle \mathbb{R}, \tau_4 \rangle$.

iii) Every locally compact space is compact.

iv) T_1 space preserved under the continuous function.

v) Fort's space is second axiom space.

vi) Any indiscrete topological space $\langle X, \tau \rangle$ is T_2-space.

vii) Fort's space is $T_3 \frac{1}{2}$ space.

P.T.O.

2. a) Prove that being T_0 space is topological property. **4**
   b) Prove that every second axiom space is first axiom space. **4**
   c) Let $A \subseteq X$ define $\tau = \{\phi\} \cup \{B \subseteq X \mid A \subseteq B\}$ then show that τ is topology on X. **3**
   d) In any topological space $\langle X, \tau \rangle$ prove that $e(E) = e[X - e(E)]$ for any $E \subseteq X$. **3**

3. a) Let C* be a closure operator defined on X. Let $F = \{F \subseteq X | C^*(F) = F\}$ and $\tau = \{X - F / F \in F\}$ then prove that τ is topology on X. and $C^*(A) = \overline{A}$ = closure of A in $\langle X, \tau \rangle$ for any $A \subseteq X$. **7**
   b) Let $\langle X, \tau \rangle$ and $\langle X^*, \tau^* \rangle$ be topological spaces and $f : X \to X^*$, then prove that f is continuous on X if and only if the inverse image of closed set in X* is closed in X. **7**

4. a) Prove that being Lindelof space is topological property. **7**
   b) Let f be a mapping of topological space $\langle X, \tau \rangle$ onto a set Y. **7**
   Define $\tau^* = \{G \subseteq Y | f^{-1}(G) \in \tau\}$ then show that :
   1) τ^* is topology on Y.
   2) $f : \langle X, \tau \rangle \to \langle Y, \tau^* \rangle$ is continuous function.
   3) τ^* is the largest topology for which $f : X \to Y$ is continuous.
   4) $F \subseteq Y$ is closed in $\langle Y, \tau^* \rangle$ if and only if $f^{-1}(F)$ is closed in $\langle X, \tau \rangle$.

5. a) Let E be the subset of the subspace $\langle X^*, \tau^* \rangle$ of topological space $\langle X, \tau \rangle$ then show that E is τ^* connected if and only if E is τ connected. **7**
   b) Prove that a topological space $\langle X, \tau \rangle$ is a T_1 space if and only if $\cap \{G \mid G \in \tau, x \in G\} = \{x\}$. **7**

6. a) Prove that any topological space is subspace of separable space. **7**
   b) Prove that any compact subset of a T_2 space is closed set. **7**

7. a) Show that open subspace of separable is separate. **7**
   b) Show that continuous image of connected space is connected. **7**

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Seat No.	

M.Sc. – I (Semester – II) (New-CBCS) Examination, 2016
MATHEMATICS
Complex Analysis (Paper – IX)

Day and Date : Wednesday, 6-4-2016

Time : 10.30 a.m. to 1.00 p.m. Max. Marks : 70

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *from Q.* ***3*** *to Q.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Attempt the following MCQs. **6**

1) If $f : G \to \mathbb{C}$ is continuous and $\int_T f = 0$ for every triangular path T then
   a) f is constant one
   b) f is identically equal to zero on G
   c) f is not differentiable on G
   d) f is analytic on G

2) The mapping $T(z) = \dfrac{az}{cz+b}$ is conformal iff
   a) $a \neq 0$ b) $ad \neq 0$ c) $d \neq 0$ d) $c \neq 0$

3) If $f(z) = z^3$ then
   a) f(z) has an essential singularity at $z = \infty$
   b) f(z) has a pole of order 3 at $z = \infty$
   c) f(z) has a pole of order 3 at z = 0
   d) f(z) is analytic at $z = \infty$

4) Let T be any circle enclosing the origin and oriented anticlockwise then the value of integral $\int \dfrac{\cos z}{z^2}\, dz$ is
   a) $2\pi i$ b) 0
   c) $-2\pi i$ d) none of these

5) If $f(z) = \frac{1}{z^4 + 1}$ then

a) f(z) has simple poles at z = $\pm$ i, $\pm$ 1

b) f(z) has poles of order 2 at z = 1, i

c) f(z) has simple poles at $z = \pm \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}} i$

d) None of these

6) If f : $\mathbb{C} \to \mathbb{C}$ is defined as f(z) = e^z then

a) f(z) is surjective but not injective

b) f(z) is injective but not surjective

c) f(z) is bijective

d) f(z) is neither surjective nor injective

B) State whether the following statements are **true** or **false**. **6**

1) All the zeros of sinz are reals.

2) The function f(z) = log z is analytic in complex plane.

3) The Mobius transformation having 3 fixed points is identity transformation.

4) Mobius transformation $T(z) = \frac{az + b}{cz + d}$ maps circles onto circles.

5) The function f(z) = $\bar{z}$ is analytic at origin.

6) The function f(z) = sin z is bounded in complex plane.

C) Define : **2**

1) Singular part.

2) Normal family.

2. a) Evaluate $\int_\gamma \frac{e^{iz}}{z^2} dz$ where $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$. **3**

b) If f is analytic in B(a ; R) and if $|f(z)| \le M$ for all z in B(a ; R) then show that

$|f^{(n)}(a)| \le \frac{n!M}{R^n}$. **4**

c) Define $\gamma(t) = t^2 + it$ $0 \le t \le 1$ is $\gamma(t)$ closed and smooth curve. **3**

d) If f has a pole of order m at z = a and $g(z) = (z-a)^m f(z)$ then show that

$$\text{Res}(f; a) = \frac{1}{(m-1)!} g^{(m-1)}(a).$$ **4**

3. a) State and prove Morera's theorem. **8**

b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$. **6**

4. a) If f is analytic in the annulus ann $(a_1 R_1 ; R_2)$ then prove that

$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$ where the convergence is absolute and uniform over ann $(a ; r_1 r_2)$ if $R_1 < r_1 < r_2 < R_2$ and the coefficients are given by

$$a_n = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{(z-a)^{n+1}} dz.$$ **8**

b) State and prove Liouville's theorem. **6**

5. a) State and prove Rouche's theorem. **7**

b) If G is an open subset of the plane and $f : G \to C$ is an analytic function and if γ is a closed rectifiable curve in G such that $n(\gamma ; w) = 0$ for all w in GG then prove that for a in $G - \{\gamma\}$ **7**

$$n(\gamma; a)\ f(a) = \frac{1}{2\Pi i} \int_{\gamma} \frac{f(a)}{z-a} dz.$$

6. a) Prove that a family F in H(G) is normal iff F is locally bounded. **8**

b) Prove that a Mobius transformation takes a circle onto circle. **6**

7. a) If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \ge |f(z)|$ for all z in G then prove that f is constant. **7**

b) Show that $\int_{0}^{\infty} \frac{x^{-c}}{1+x} dx = \frac{\pi}{\sin \pi c}$ if $0 < c < 1$. **7**

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SLR-MA – 521

Seat No.	

M.Sc. – I (Semester – II) (New CBCS) Examination, 2016
MATHEMATICS
Relativistic Mechanics (Paper No. X)

Day and Date : Saturday, 9-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :
   1) In the process of contraction, the rank of tensor reduces by _________
   2) The transformation equation for charge density is $\rho' =$ _________
   3) The frames relative to which an unaccelerated body appears accelerated are called _________
   4) Galilean transformations in vector form are given by ________
   5) A moving clock runs ________ than a stationary one.
   6) Kinetic energy of a moving body is equal to ________ times square of speed of light.
   7) The motion of one projectile as seen from another projectile will always be a _________ motion.

   B) State whether the following statements are **true** or **false** (**one** mark **each**) :
   1) Force is the source by which the state of body whether in motion or at rest may be changed.
   2) The composition of two velocities which are separately less than C, will always exceed the velocity of light.
   3) In elastic collision the bodies after collision, coalesce and move with common velocity.
   4) Aberration is something that deviates from normal way.
   5) All inertial frames are physically equivalent.
   6) In all types of collision, the total energy is always conserved.
   7) Transverse Doppler effect is purely relativistic.

P.T.O.

2. a) Explain why gravity is ignored in the special theory of relativity ?
   b) Give reasons that the velocity of light is called fundamental velocity.
   c) The length of a rocket ship is 100 metres on the ground. When it is in flight its length observed on the ground is 99 metres. Calculate its speed.
   d) Prove that for finite velocities an expression for relativistic kinetic energy reduces to classical expression. **(3+3+4+4)**

3. a) Prove or disprove $\nabla^2\phi = 0$ is invariant under Lorentz transformation.
   b) State postulates of special theory of relativity and hence deduce Lorentz transformation. **(7+7)**

4. a) Determine the relativistic equation for the Doppler effect in the form

$$\gamma = \gamma'\left(\frac{1+\beta\cos\theta'}{\sqrt{1-\beta^2}}\right).$$

   b) Obtain the law of relativistic force. **(7+7)**

5. a) Derive the transformation equations for electric field.
   b) Prove that : A set of quantities whose outer product with an arbitrary tensor gives a tensor then the quantity itself is tensor. **(7+7)**

6. a) Show that the circle $x^2 + y^2 = a^2$ in a frame S appears to be an ellipse in S' which is moving with velocity v relative to S along X-axis.
   b) Obtain the relativistic expression for Lagrangian. **(7+7)**

7. a) Prove that : The following Maxwell's equations are invariant under Lorentz transformation $\partial iv\bar{B} = 0$, curl$\bar{E} = \frac{-\partial\bar{B}}{\partial t}$.
   b) Obtain the relativistic expression $m = \frac{m_0}{\sqrt{1-4^2/c^2}}$ for the mass. **(7+7)**

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Seat No.	

M.Sc. – I (Semester – II) (Old) (CGPA) Examination, 2016
MATHEMATICS (Paper – VI)
Algebra – II

Day and Date : Wednesday, 30-3-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to ***right*** *indicates* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) : **8**

1) An algebraic number 'a' is said to be ____________ if it satisfies an equation of the form $\alpha^m + \alpha_1 a^{m-1} + \alpha_2 a^{m-2} + ... + \alpha_m = 0$ where $\alpha_1, \alpha_2 .... \alpha_m$ are integers.

2) There is one-one correspondence between the subfields of splitting field (containing the given field) of f(x) and the subgroups of its __________

3) The complex number w is a ___________ n^{th} root of unity if $w^n = 1$ but $w^n \neq 1$ for $0 < m < n$.

4) E is splitting field of f(x) over F if E is ___________ extension of F in which f(x) has all its roots.

5) If $a \in k$ is algebraic of degree n over F then $[F(a) : F] =$ _________

6) If Q is the set of rational numbers then the set $Q\left(\sqrt{3}, \sqrt{5}\right)$ is _________

7) If k be an extension of the field F and $a, b \in k$ be algebraic over F of degree m and n respectively then a – b is algebraic over F of degree _________

8) A field F is said to be algebraically closed if it has _________

P.T.O.

B) State **true** or **false** : **6**

1) The set of algebraic numbers forms a field.

2) The general poly of degree $n \geq 5$ is not solvable by radicals.

3) Every algebraic extension of a field is a finite extension.

4) For all $\alpha, \beta \in F$ there is always an automorphism of E mapping α on to β.

5) Q(i) is a splitting field over Q.

6) C is simple extension of R.

2. a) Prove that the mapping $\psi : F[x] \to F(a)$ defined by $\psi(h(x)) = h(a) \forall h(x) \in F[x]$ is a homomorphism. **3**

b) Define :

i) Field extension

ii) Algebraic extension

iii) Normal extension. **3**

c) Determine the degree of splitting field of the polynomial $x^4 - 2$ over the field of rational numbers. **4**

d) If K is normal extension of field of characteristic O then prove that [K : F] = O (G (K, F)). **4**

3. a) Prove that A field of characteristic 'O' is perfect. **6**

b) Let ψ be an isomorphism of a field F onto field F' defined by $\psi(\alpha) = \alpha'$ for every $\alpha \in F$. For an arbitrary polynomial, $f(x) = \alpha_0 + \alpha_1 x + ... + \alpha_n x^n \in F[x]$ Let us define,

$$f'(t) = \alpha'_0 + \alpha'_1.t + ... + \alpha'_n.t^n \in F'[t]$$

If f(x) is irreducible in F(x) then prove that there exists an isomorphism θ of $\frac{F[x]}{\langle f(x) \rangle}$ onto $\frac{F'[t]}{\langle f'(t) \rangle}$ such that $\theta(\alpha) = \psi(\alpha) = \alpha' \forall \alpha \in F$. **8**

4. a) If p(x) is an irreducible polynomial in F(x) of degree $n \geq 1$ then prove that there is an extension E of F such that [E : F] = n in which p(x) has a root. **7**

   b) If R be the field of real numbers and Q be the field of rational numbers then show that $\sqrt{2}+\sqrt[3]{5}$ is algebraic over Q of degree 6. **7**

5. a) P.T. the poly $f(x) \in F[x]$ has a multiple root iff f(x) and $f'(x)$ have a non-trivial common factor. **7**

   b) If a real number α is constructible then prove that α lies in some finite extension K of the field of rational numbers of degree which is some power of 2. **7**

6. a) Find the Galois group of $x^2 - 2$ over the field of rational numbers. **7**

   b) Prove that : A field K is a normal extension of a field F of characteristic O iff K is a splitting field of some polynomial over F. **7**

7. a) Prove that for every prime number p and every positive integer m there exists a field having p^m elements. **7**

   b) IF E is an extension field of F and $\alpha \in E$ be algebraic of odd degree over F then show that α^2 is algebraic of odd degree over F. **7**

SLR-MA – 523

Seat No.	

M.Sc. – I (Semester – II) Examination, 2016
MATHEMATICS (Old) (CGPA)
Real Analysis – II (Paper – VII)

Day and Date : Friday, 1-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *i) Q. No. **1** and **2** are **compulsory**.*
*ii) Attempt **any three** questions from Q. No.**3** to Q. No.**7**.*
*iii) Figures to the **right** indicate **full** marks.*

1. a) Fill in the blanks : **7**

i) The negative part f^- of a function f is given by $f^-(x)$ = __________

ii) If E_1 and E_2 are measurable sets then $m^*(E_1 \cup E_2) + m^*(E_1 \cap E_2)$ = __________

iii) If f and g are bounded measurable functions defined on the set E of finite measure and if a are b any constants then $\int_E (af + bg)$ = ______________

iv) If A and B are any two sets then $\chi_{A \cap B}$ = ______________

v) If φ is a simple function defined on any measurable set E then $\int_E \varphi$ = _________

vi) If f is absolutely continuous on [a, b] and if $f'(x) = 0$ a.e. then f is ___________

vii) If $a \le c \le b$ then $T_a^b(f)$ = ____________

b) State whether following is **True** or **False** : **7**

i) If f and g are bounded measurable functions defined on a set E of finite measure. If $f \le g$ a.e. then $\int_E g \le \int_E f$.

ii) A non-negative measurable function f is integrable over the measurable set E if $\int_E f = \infty$

P.T.O.

iii) The upper left-hand derivative of the function f at x is given by $D^{-}f(x) = \overline{\lim_{h \to 0^+}} \frac{f(x) - f(x-h)}{h}$

iv) If ϕ is a continuous function on (a, b) and if $D^+ \varphi$ is non-decreasing, the φ is convex.

v) If f^+ and f^- are the positive and negative parts of the function f respectively then $f = f^+ + f^-$.

vi) Integral of simple function is greater than or equal to zero.

vii) If $T_a^b (f) < \infty$, then f is of bounded variation over [a, b].

2. a) State Egoroff's theorem. .3

b) If m* (E) = 0 then prove that E is measurable. 3

c) If f and g are two measurable real-valued functions defined on the same domain then prove that f + g is also measurable. 4

d) If f (x) = – g(x) then prove that D+f(x) = – D – g (– x). 4

3. a) If $E \subseteq [0,1)$ is measurable set, then prove that for each $y \in [0,1)$ then E + y is measurable and m (E + y) = m (E). 7

b) If (f_n) is a sequence of measurable functions defined on a set E of finite measure, and suppose that there is a real number M such that $|f_n(x)| \leq M$ for all n and for all x. If $f(x) = \lim_{n \to \infty} f_n(x)$ for each $x \in E$, then prove that $\int_E f = \lim_{n \to \infty} \int_E f_n$. 7

4. a) If E_1 and E_2 are measurable sets then prove $E_1 \cup E_2$ is also measurable set. 7

b) If a function f is integrable on [a, b], then prove that the function F defined by $F(x) = \int_a^x f(t)dt$ is a continuous function of bounded variation on [a, b]. 7

5. a) Prove that a function f is of bounded variation on [a, b] if and only if the function f is the difference of two monotone real-valued functions on [a, b]. **7**

   b) If f is a measurable function and f = g a.e. then prove that the function g is measurable. **7**

6. a) If $\{A_n\}$ is a countable collection of sets of real numbers, then prove that

   $$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \le \sum_{n=1}^{\infty} m^*(A_n).$$ **7**

   b) Prove that a function F is an indefinite integral if and only if it is absolutely continuous. **7**

7. a) If φ is continuous function on (a,b) and if $D^+ \varphi$ is nondecreasing, then prove that φ is convex. **7**

   b) If $\langle f_n \rangle$ is a sequence of nonnegative measurable functions and $f_n(x) \to f(x)$ almost everywhere on a set E, then prove that $\int_E f \le \underline{\lim}_{n \to \infty} \int_E f_n$. **7**

SLR-MA – 524

Seat No.	

M.Sc. – I (Semester – II) Examination, 2016
MATHEMATICS (Old) (CGPA)
General Topology (Paper No. – VIII)

Day and Date : Monday, 4-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) State the followings either **True** or **False** : **10**

1) The intersection of a finite family of neighbourhoods of a point in a topological space X is a neighbourhood of the point.
2) Union of two topologies for X is also a topology for X.
3) Separability of a topological space is a hereditary property.
4) Every discrete space is not a first countable.
5) Every co-finite topological space is compact.
6) Confinite topological space X is connected if X is infinite.
7) Every singleton set is always connected.
8) Any closed interval [a, b] is homeomorphic to [0, 1].
9) If f is a homeomorphism then f is only continuous.
10) Composition of two continuous mappings is also a continuous map.

B) Fill in the blanks : **4**

11) $(\overline{A}) =$ ______________.
12) A topological space X is said to be T_1 if ______________.
13) Two non empty subsets A and B of a space X are said to be separated if ______________.
14) A subset A of a topological space X is said to be compact if ________.

P.T.O.

2. a) If X = {a, b, c} and $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ then show that (X, τ) is a regular space. **3**

   b) Show that usual topological space is a separable. **3**

   c) Show that every discrete topological space is T_0-space. **4**

   d) Show that every metric space is T_2-space. **4**

3. a) Prove that continuous image of connected space is connected. **7**

   b) Prove that a topological space X is disconnected if and only if there exists a non empty proper subset of X which is both open and closed in X. **7**

4. a) Prove that a topological space X is compact if and only if every basic open cover of X has a finite subcover. **7**

   b) Prove that continuous image of compact space is compact. **7**

5. a) Prove that every second countable space is a Lindelof space. **7**

   b) Prove that a topological space (X, τ) is a T_0-space if and only if for any distinct arbitrary points x, y of X, the closure of {x} and {y} are distinct. **7**

6. a) Prove that (X, τ) is a Hausdorff topological space if and only if the intersection of all closed neighbourhoods of each point of X is a singleton. **7**

   b) Prove that IR is connected. **7**

7. a) Prove that every compact Hausdorff space is regular. **7**

   b) Prove that every regular Lindeloff space is a normal. **7**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2016
MATHEMATICS (Old) (CGPA)
Complex Analysis (Paper – IX)

Day and Date : Wednesday, 6-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No. **1** and Q. No. **2** are **compulsory**.*
*2) Solve **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct answer (**one** mark each) : **6**

1) The radius of convergence for the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is = ______________

a) 0 b) 1
c) Finite d) ∞

2) The value of the integral $\int_r \frac{e^{iz}}{z^2} dz$, where r : |z| = 1 is ______________

a) -2π b) 2π
c) $2\pi i$ d) 0

3) Suppose f has a pole of order m at z = a and put g(z) = $(z - a)^m$.f(z) then Res(f;a) = ____________

a) $\lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m \cdot f(z)\right]$

b) $\frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m \cdot f(z)\right]$

c) $\lim_{z \to a} \frac{d^{m}}{dz^{m}}\left[(z-a)^m \cdot f(z)\right]$

d) Zero

4) Let f be analytic in the disk B(a;R) and suppose that r is a closed rectifiable curve in B(a;R) then

a) $\int_r f > 0$ b) $\int_r f < 0$

c) $\int_r f = 0$ d) $\int_r f = 1$

5) Which of the following is an analytic function ?

a) sin z b) cos z c) e^z d) all of three

6) Let $f : G \to \mathbb{C}$ be analytic and $\overline{B}(a;r) \subset G(r>0)$. If $v(t) = a+re^{it}$, $0 \le t \le 2\pi$ then for $|z-a|<r$, f(z) = __________

a) $\frac{1}{2\pi i}\int_r \frac{f(w)}{(w-z)}\,dw$

b) $\frac{1}{2\pi !}\int_r \frac{f(w)}{(w-z)^n}\,dz$

c) $\frac{n!}{2\pi i}\int_r \frac{f(w)}{(w-z)}\,dw$

d) $\frac{n!}{2\pi i}\int_r \frac{f(w)}{(w-z)^n}\,dz$

B) Fill in the blanks (**one** mark **each**) : **5**

1) The z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$ then (z_1, z_2, z_3, z_4) is a real number if and only if all four points lie's on ____________

2) If f is an analytic on region G, $a \in G$, f(a) = 0 then z = a is a zero of f of multiplicity four if ___________

3) If f is an analytic at point z = a then the point z = a is called as __________ point.

4) The complex valued function $\frac{\sin z}{z}$ has a __________ singularity at z = 0.

5) A polynomial $p(z) = z^n + a_{n-1}z^{n-1} + ... + a_1z + a_0$ has exactly __________ roots in $\mathbb{C}$.

C) Define the terms : **3**

1) Entire function

2) Pole

3) Cross Ratio.

2. a) If f is a bounded entire function then prove that f is a constant. **4**

b) Show that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} = \frac{\pi}{6}$. **3**

c) Find the Mobius transformation that maps 0, 1, – 1 onto i, 2, 4 respectively. **3**

d) State and prove argument principle. **4**

3. a) Let f be an analytic function in the region G except for the isolated singularities $a_1, a_2, ..., a_m$. If r is a closed rectifiable curve in G which does not pass through any of the points a_k and $r \approx 0$ in G then prove that

$$\frac{1}{2\pi i}\int_r f = \sum_{k=1}^{m} n(r;a_k)\mathrm{Res}(f;a_k).$$ **7**

b) Find the Laurent'z expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in the ann(0; 0,1). **7**

4. a) If r : [0, 1] $\rightarrow \mathbb{C}$ is a closed rectifiable curve and $a \notin \{r\}$ then prove that

$\frac{1}{2\pi i}\int_r \frac{dz}{z-a}$ is an integer. **6**

b) Let G is an open set in $\mathbb{C}$. If f : G $\rightarrow$ $\mathbb{C}$ be a differentiable then show that f is analytic on G. **8**

5. a) Let f to be analytic in B(a;R) with f(a) = 0. Show that a is a zero of multiplicity m iff f(a) = 0 = f'(a) = ...= $f^{(m-1)}(a)$ amd $f^{(m)}(a) \neq 0$. **7**

b) State and prove open mapping theorem. **7**

6. a) Let G be an open subset of the plane and $f : G \to \mathbb{C}$ is an analytic function. If $r_1, r_2, \ldots r_m$ are closed rectifiable curves in G such that

$n(r_1; w) + n(r_2; w) + \ldots + n(r_m; w) = 0$ for all $w \in \mathbb{C} - G$ then for $a \in G - \bigcup_{j=1}^{m} \{r_j\}$

and $k \geq 1$, $f^{(k)}(a) \sum_{j=1}^{m} n(r_j; a) = k! \sum_{j=1}^{m} \frac{1}{2\pi i} \int_{r_j} \frac{f(w)\,dw}{(w-a)^{k+1}}$. **8**

b) Let G be a region in $\mathbb{C}$ and f is an analytic function on G. Suppose, there is a constant M such that $\limsup_{z \to a} |f(z)| \leq M$, for all $a \in \partial_\infty G$. Then prove that $|f(z)| \leq M$, $\forall z \in G$. **6**

7. a) State and prove the Riemann mapping theorem. **8**

b) Evaluate $\int_{|z|=1} \frac{e^z - 2}{e^z - 2z - 1} dz$ by using Rouche's theorem. **6**

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SLR-MA – 526

Seat No.	

M.Sc. I (Semester – II) (Old) (CGPA) Examination, 2016
MATHEMATICS
Relativistic Mechanics (Paper No. – X)

Day and Date : Saturday, 9-4-2016 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

N.B. : *1) Q. No. **1** and **2 compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark each) : **10**

1) A region where in a small magnet or a loop of wire carrying current will experience a force is called ____________
2) The components of a contravariant vector are actually the components of a contravariant tensor of ______________
3) Mass of body ____________ with increase of velocity.
4) Like classical Hamiltonian relativistic Hamiltonian also denotes the __________ of the system.
5) The frames with respect to which an unaccelerated body appears unaccelerated are ____________
6) Einstein has developed a special theory of relativity in ____________
7) The transformation equation for mass is given by m = ______________
8) If v<<<c then $u' \oplus v$ = _________________
9) The relativistic transverse Doppler effect is given by γ = ____________
10) The addition of any velocity of light to velocity of light merely reproduces the ____________

P.T.O.

B) State **True** or **False** (**one** mark each) : **4**

1) The acceleration observed by the observer in different inertial frames is same (under G.T.).

2) The conservation of classical momentum in the collision of two particles is invariant under Lorentz transformations.

3) The four momentum is like a time like vector.

4) A moving clock appears to run at its fastest rate.

2. a) What is a twin paradox in special relativity ? **3**

b) Show that the circle $x^2 + y^2 = a^2$ in a frame S appears to be an ellipse in S' which is moving with velocity V relative to S along X-axis. **4**

c) A body has dimension $8i + 3j + 5k$ in S'-frame. How these dimensions will be represented in S-frame if S' is moving with velocity 0.8 C along $X - X'$ axis ? **4**

d) Explain Doppler effect with examples. **3**

3. a) Prove that the three dimensional volume element dxdydz is not invariant but four dimensional volume element dxdydzdt is invariant under Lorentz transformations. **7**

b) With usual notations show that,

$$\frac{c^2 - u'^2}{c^2} = \frac{(c^2 - u^2)(c^2 - v^2)}{(c^2 - u_x v)^2}$$ **7**

4. a) A scientist observes that a certain atom 'A' moving relative to him with velocity 2×10^{10} cm/s emits a particle B which moves with velocity 2.8×10^{10} cm/s with respect to atom. Calculate the velocity of the emitted particle relative to scientist. **6**

b) Derive the law of relativistic force. **8**

5. a) Obtain the relativistic aberration formula from the velocity transformation equations. **7**

b) Show that Lorentz transformations forms a group. **7**

6. a) Show that the Newton's laws of motion are invariant under Galilean transformations. **7**

b) Derive the transformation equations for charge density of current density. **7**

7. a) Obtain Lorentz's velocity transformation equations. **7**

b) A covariant tensor Si has components (1, 3, 7) in a rectangular cartesian co-ordinates. Find its components in spherical polar co-ordinates. **7**

SLR-MA – 527

Seat No.	

M.Sc. – II (Semester – III) (New CGPA) Examination, 2016
MATHEMATICS
Functional Analysis (Paper No. – XI)

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

***Instructions** : i) Q. No. **1** and **2** are **compulsory**.*
*ii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iii) Figures to the **right** indicate **full** marks.*

1. a) Fill in the blanks : **7**

i) The elements of N*, the conjugate space of the normed linear space N are called _________

ii) The conjugate space of l_1^n is _________

iii) If N is a non-zero normed linear space then N is Banach space if and only if the set {x : ||x|| = 1} is _________

iv) A closed linear subspace M of a Hilbert space H is said to be invariant under T if __________

v) If we define $\| x \|_p = \left\{ \sum_{i=1}^{n} | x_i |^p \right\}^{\frac{1}{p}}$ then the Holder's inequality can be written as ________.

vi) If H is a complex inner product space then for all x, y, $z \in H$ we have $(x, \alpha y + \beta z) =$ ___________ .

vii) If $\{x_n\} \to x$ and $\{y_n\} \to y$ in a normed linear space N then $\{x_n + y_n\} \to$ ________

P.T.O.

b) State whether following is **true** or **false** : 7

i) The operations of addition and scalar multiplication in a normed linear space N are not jointly continuous.

ii) If N is a normal operator on a Hilbert space H then $||N^2|| = ||N|^2$.

iii) If T is an operator on a Hilbert space H for which (Tx, x) = 0 for all $x \in H$ then T = 0.

iv) A normed linear space is a Banach space.

v) If x and y are two vectors in a Hilbert space H then
$||x + y||^2 + ||x - y||^2 = ||x||^2 + ||y||^2$.

vi) If $\{e_i\}$ is an orthonormal set in a Hilbert space H then $\{e_i\}$ is complete if and only if $x \perp \{e_i\} \Rightarrow x = 0$.

vii) If a normed linear space N is a metric space with respect to the metric defined by $d(x, y) = ||x - y||$ for every $x, y \in N$ then $d(x + z, y + z) = d(x, y)$ for all $x, y, z \in N$.

2. a) If a one-to-one linear transformation T of a Banach space onto itself is continuous then prove that its inverse T^{-1} is continuous. 3

b) If x and y are any two orthogonal vectors in a Hilbert space H then prove that $||x + y||^2 = ||x - y||^2 = ||x||^2 + ||y||^2$. 4

c) If $T, T' \in \mathcal{B}(N)$ and if $T_n \to T$ and $T_n' \to T'$ then prove that $T_n T' \to TT'$ as $n \to \infty$. 4

d) If H is finite-dimensional Hilbert space H, then show that every isometric isomorphism of H into itself is unitary. 3

3. a) If N is normed linear space and x_0 is non-zero vector in N, then prove that there exists a functional f_0 in N* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$. 7

b) Prove that a non-empty subset X of normed linear space N is bounded if and only if f(X) is a bounded set of numbers for each f in N*. 7

4. a) Let B be a Banach space and N a normed linear space. If $\{T_n\}$ is a sequence in $\mathcal{B}(B, N)$ such that $T(x) = \lim_{n \to \infty} T_n(x)$ exists for each x in B, then prove that T is a continuous linear transformation. 7

b) If P is a projection on a Banach space B and if M and N are its range and null spaces respectively, then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$. 7

5. a) If x, y are any two vectors in a Hilbert space H then prove that
$4(x, y) = ||x + y||^2 - ||x - y||^2 + i||x + iy||^2 - i|| x - iy||^2$. **7**

b) If M is proper closed linear subspace of a Hilbert space H, then prove that there exists a non-zero vector z_0 in H such that $z_0 \perp M$. **7**

6. a) If $\{e_i\}$ is an orthonormal set in a Hilbert space H and if x is any vector in H, then prove that the set $S = \{e_i : (x, e_i) \neq 0\}$ is either empty or countable. **7**

b) If A is a positive operator on H, then prove that I + A is non-singular. Also show that I + T*T and I + T T* are non-singular for arbitrary operator T on H. **7**

7. a) If T is a contraction defined on a complete metric space X, then prove that T has a unique fixed point. **7**

b) If S is a non-empty subset of a Hilbert space H, then show that the set of all linear combinations of vectors in S is dense in H if and only if $S^{\perp} = \{0\}$. **7**

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Seat No.	

M.Sc. – II (Semester – III) (New) (CGPA) Examination, 2016
MATHEMATICS
Advanced Discrete Mathematics (Paper – XII)

Day and Date : Thursday, 31-3-2016 Total Marks : 70
Time :2.30 p.m. to 5.00 p.m.

Instructions *: 1) Figures to the* ***right*** *indicate* ***full*** *marks.*
2) Q. No. ***1*** *and* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*

1. A) Fill in the blanks. (**one** mark **each**) **7**

i) A graph without any loops and parallel edges is called as __________

ii) Any totally ordered set is __________

iii) The minimum number of edges in a connected graph with n vertices is __________

iv) The vertex of degree zero is called __________

v) An expression for geometric series $\frac{1}{1+ax}$ is __________

vi) $K_{m,n}$ is regular graph if __________

vii) The Graph $K_{l,n}$ is called __________

B) State **true** or **false** : (**one** mark **each**) **7**

i) Every complete graph is regular.

ii) Every connected graph is tree.

iii) Every order preserving function is lattice homomorphism.

iv) Every distributive lattice is a modular lattice.

v) Every walk is a path.

vi) A complete graph with n vertices have n(n – 1) edges.

vii) A connected graph with n vertices and (n – 1) edges is a tree.

2. a) Define isomorphism of graph with two examples. **(4+3+4+3)**
   b) Show that every totally ordered set is lattice.
   c) Show that every Boolean ring is a commutative ring.
   d) Write the short note on fusion of two vertices in a graph.

3. a) State and prove bridge theorem. **(7+7)**
   b) Let (B, +, $*$, 0, 1) is a Boolean ring. Then show that (B, $\wedge$, $\vee$, 0, 1) is a Boolean ring.

4. a) Give the short note on matrix representation of graph with two examples. **(7+7)**
   b) Show that a graph G is connected if and only if it has a spanning tree.

5. a) Determine the number of integers between 1 to 2000 both inclusive which are divisible by 10, 11, 12. **(7+7)**
   b) Find the general solution of $a_r - 5a_{r-1} + 6a_{r-2} = 8r + 5$.

6. a) Calculate the generating function of $\frac{1}{1-5x+6x^2}$. **(7+7)**
   b) Let L and L′ be any two lattices. A bijective function f : L $\to$ L′ is lattice isomorphism if and only if both f and f^{-1} preserves order.

7. a) Show that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G. **(7+7)**
   b) Show that the lattice of submodules of a module is a modular lattice.

SLR-MA – 529

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2016
MATHEMATICS (Paper – XIII)
Elective – I : Linear Algebra (New) (CGPA)

Day and Date : Saturday, 2-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions *: 1) Q.No.* ***1*** *and Q.No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q.No.* ***3*** *to Q.No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**). **5**

i) Let V and W be inner product spaces over the same field, and let T be a linear transformation from V into W then T preserves distances if for all α, β in V;

$\| T(\alpha) - T(\beta) \| =$ __________

ii) If W be a subspace of a vector space V (F), then A (W) = ______________, where A(W) is an annihilator of W.

iii) Let $W_1, W_2, ..., W_k$ be subspaces of the vector space V (F), we say that $W_1, W_2, ..., W_k$ are independent if $\alpha_1 + \alpha_2 + ... + \alpha_k = 0, \alpha_i \in W_i$; for i = 1, 2, ..., k then ______________

iv) A complex $n \times n$ matrix A is called unitary if ______________

v) The linear operator T is called triangulable if there is an ordered basis in which T is represented by a ______________ matrix.

B) State whether the following are **true** or **false** (**one** mark **each**). **5**

i) Let V be a vector space over F and T is a linear operator on V. If zero is an eigen value of T then T is non singular.

ii) A linear operator T on a finite dimensional vector space V (F) is a diagonalizable if V is the direct sum of the eigen spaces of T.

iii) Let λ be an eigenvalue of a linear operator T on V (F). Then the algebric multiplicity of λ does not exceed its geometric multiplicity.

iv) If dim V (F) = n then dim V* (F) = n^2, where V* is the dual of V.

v) The minimal polynomial of a matrix is unique.

P.T.O.

C) Choose the correct alternative (**one** mark **each**). **4**

i) The characteristic polynomial of a matrix $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ is

a) $(x - 5)(x - 3)$
b) $(x - 4)(x - 3)(x - 5)$
c) $(x + 4)(x + 3)(x + 5)$
d) $(x - 12)$

ii) Let T be a linear operator on a finite dimensional complex inner product space V (F). Then

I) T is normal if the adjoint T* of T is a polynomial in T.
II) The adjoint T* of T is a polynomial in T if T is normal.

a) Only I is true
b) Only II is true
c) Both I and II are true
d) Both I and II are false

iii) If A is an $n \times n$ nilpotent matrix of index k then minimal polynomial of A is

a) x
b) x^{k-1}
c) x^{k+1}
d) x^k

iv) Let A be an $n \times n$ matrix with characteristic polynomial $f(x) = (x - c_1)^d (x - c_2) \ldots (x - c_k)$ then trace (A) is

a) $c_1 + c_2 + \ldots + c_k$
b) zero
c) c_1
d) none of these

2. a) Let dim V (F) = n and let $1B = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis for V. Then show that there is a uniquely determined a basis $1B^* = \{f_1, f_2, \ldots, f_n\}$ for V* such that $f_i(\alpha_j) = \delta_{ij}$ where $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$. **4**

b) Let dim V (F) is finite and T be a linear operator on V. Then show that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2) \ldots (x - c_k)$ where $c_1, c_2, \ldots, c_k$ are distinct elements of F. **4**

c) If the characteristic polynomial of a 5×5 matrix A is $(x - 3)^3 (x - 2)^2$ then find all possible Jordan canonical forms of A. **3**

d) Let U be a linear operator on an inner product space V. Then show that U is unitary if and only if the adjoint U* of U exists and U*U = UU* = I. **3**

3. a) Let dim V(F) is finite and W be a subspace of V. Then show that dim W + dim A (W) = dim V. **7**

b) Let dim V (F) = n and IB = $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a basis for V. If $\{x_1, x_2, ..., x_n\}$ is any set of n scalars then show that there exists a unique linear functional f on V. Such that $f(\alpha_i) = x_i$, for all i, i = 1, 2, ..., n. **7**

4. a) State and prove Cayley-Hamilton theorem. **7**

b) State and prove primary decomposition theorem. **7**

5. a) Let V be a finite dimensional vector space. Let $W_1, W_2, ..., W_k$ be subspaces of V and let $W = W_1 + W_2 + ... + W_k$. Then show that the following are equivalent.

i) $W_1, W_2, ..., W_k$ are independent.

ii) For each j, $2 \leq j \leq k$, we have $W_j \cap (W_1 + ... + W_{j-1}) = \{0\}$.

iii) If IB_i is an ordered basis for W_i, $1 \leq i \leq k$, then the sequence IB = $(IB_1, IB_2, ..., IB_k)$ is an ordered basis for W. **7**

b) Find a 3×3 matrix for which characteristic polynomial is x^2. Then find the minimal polynomial of corresponding matrix. **7**

6. a) Find the minimal and the rational form of the following real matrix

$$\begin{bmatrix} c & 0 & -1 \\ 0 & c & 1 \\ -1 & 1 & c \end{bmatrix}.$$ **7**

b) Determine all possible Jordan Canonical forms for a linear operator $T : V \rightarrow V$ whose characteristic polynomial is $\Delta(t) = (t-2)^3 (t-5)^2$. **7**

7. a) Prove T is normal if and only if $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operators which commutes. **7**

b) Let V be a complex vector space and f a form on V such that $f(\alpha, \alpha)$ is real for every α. Then show that f is Hermitian. **7**

SLR-MA – 530

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
MATHEMATICS (Paper – XIV) (New) (CGPA)
Elective – II : Differential Geometry

Day and Date : Tuesday, 5-4-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions *: 1) Figures to the **right** indicate **full** marks.*

*2) Q. no. **1** and **2** are **compulsory**.*

*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**one** mark **each**) :

   1) The normal plane of $\beta(s)$ is the plane perpendicular to __________
   2) In the surface of rotation the points of the surface which lie on the axis are called __________.
   3) Directional derivative of any function f can also be expressed in terms of its __________.
   4) A point $P \in M$ is called umbilic if __________.

   B) State **true** or **false** (**one** mark **each**) :

   1) The surface northern hemisphere is a simple surface.
   2) Let α be a regular curve and β be its unit reparametrization of α then Frenet apparatus for both the curves are same with change of parameter.
   3) A surface M is called minimal surface if Guassian curvature is zero.
   4) Exterior derivative of any p-form is a p-form.

   C) Define the following terms (**two** marks **each**) :

   i) Shape operator.
   ii) Simple surface.
   iii) Proper patch.

2. a) For any function f show that d(df) = 0. **3**

b) Find the arc of a circle α(t) = (a cost, asint, 0), $0 \le t \le 2\pi$. **3**

c) If a vector field Y on a curve has constant length then prove that Y and Y' are orthogonal at each point. **4**

d) If F and G are isometries of E^3 then show that the composite mapping GF is an isometry. **4**

3. a) If U_1, U_2, U_3 are natural frame fields at P then show that $U_1[f] = \frac{\partial f}{\partial x}$, $U_2[f] = \frac{\partial f}{\partial y}$, $U_3[f] = \frac{\partial f}{\partial z}$. **7**

b) Compute Frenet apparatus of the curve $\beta(s) = \left(\frac{4}{5}\cos s,\ 1 - \sin s,\ -\frac{3}{5}\cos s\right)$. **7**

4. a) Prove that every isometry of E^3 is uniquely described as orthogonal transformation followed by translation. **7**

b) Find tangent and normal vector field on M : z = xy. **7**

5. a) Find Guassian curvature of an ellipsoid $M : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **7**

b) Show that the shape operator describes the cylindrical surface as half flat and half round. **7**

6. a) If β is reparametrization of α by h then prove that $\beta'(s) = h'(s)\alpha'(h(s))$. **7**

b) If ϕ is a 1-form on E^3 then prove that $\phi = \sum_i f_i\, dx_i$ where $f_i = \phi(U_i)$. **7**

7. a) Let v and w be tangent vectors at the same point P. Prove that v × w is orthogonal to both v and w and has length $\|v \times w\|^2 = (v.v)(w.w) - (v.w)^2$. **7**

b) Compute a proper patch in a unit sphere covering the neighbourhood of the north pole. **7**

SLR-MA – 531

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
MATHEMATICS (Paper – XV) (Elective – III)
Numerical Analysis (New) (CGPA)

Day and Date : Thursday, 7-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**) : **6**

1) Simpson's 3/8 rule for integration gives exact result when f(x) is polynomial of degree

a) 3 b) at least 3 c) at most 3 d) none of these

2) Which of the following is correct ?

a) $\nabla - '\nabla = \Delta\nabla$ b) $\nabla - \Delta = -\Delta\nabla$

c) $\nabla + \Delta = -\Delta\nabla$ d) $\nabla + \Delta = \Delta\nabla$

3) Power method is used to find

a) eigen value and corresponding eigen vector

b) largest eigen value and corresponding eigen vector

c) eigen value only

d) none of these

4) The best approximate value of the number $1/3$ is

a) 0.30 b) 0.33 c) 0.34 d) None

5) First approximation to the root of the equation $x^3 - 2x - 5 = 0$ using method of false position is

a) 2.05882 b) 2.5882 c) 2.15882 d) 2.882

6) If f(0) = 1, f(1) = 3 and f(3) = 55 then the Lagrange fundamental polynomial is

a) $(1/3)(x^2 - 4x + 3)$ b) $x^2 - 4x - 3$

c) $(1/2)(3x - x^2)$ d) $(1/6)(x^2 - x)$

P.T.O.

B) Fill in the blanks (**one** mark **each**) : **6**

1) The error in Simpson's $\frac{1}{3}$ rule over $[x_0, x_2]$ is __________

2) In Newton Raphson method the iterative formula to find $\frac{1}{N}$ is given by ________

3) The process of computing the value of the function inside the given range is called __________

4) The forward difference operator is __________

5) The method of false position is also known as _________

6) If A is diagonal matrix with diagonal elements a_{ij} then A^{-1} is also a diagonal matrix with diagonal elements __________

C) State true or false (**one** mark **each**) : **2**

1) $E = (1 - \nabla)^{-1}$

2) If A is upper triangular matrix then A^{-1} is lower triangular matrix.

2. a) Prove that $\mu^2 = 1 + \frac{1}{4}\delta^2$. **3**

b) Show that

$$e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{z!}\Delta^2 u_0 + \ldots \right) = u_0 + u_1 x + u_2 \frac{x^2}{z!} + \ldots.$$ **3**

c) The function y = sinx is tabulated below :

x	0	$\pi/4$	$\pi/2$
$y = \sin x$	0	0.70711	1.0

Using Lagranges interpolation formula, find value of $\sin(\pi/6)$. **4**

d) Prove that $\Delta = \nabla E = \delta E^{1/2}$. **4**

3. a) Describe the error in Lagranges interpolation formula. **7**

   b) Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$. **7**

4. a) Solve by using Euler's method $y' = -y$, $y(0) = 1$. **7**

   b) Derive Newton's general interpolation formula with divided differences. **7**

5. a) Prove that Newton-Raphson method converges quadratically. **7**

   b) Solve $\int_0^1 \frac{1}{1+x}\,dx$ by trapezoidal rule with h = 0.25, correct to three decimal places. **7**

6. a) Find Newtons backward difference interpolation formula. **7**

   b) Find the largest eigen value in modulus and the corresponding eigen vector of

$$\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$ using the power method. **7**

7. a) Determine the largest eigen value and the corresponding eigen vector of the matrix. **7**

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

   b) In the table below the values of y are consecutive terms of a series of which the number 21.6 is the 6^{th} term. Find the tenth term of the series. **7**

x	3	4	5	6	7	8	9
y	2.7	6.4	12.5	21.6	34.3	51.2	72.9

Using Newtons forward difference interpolation formula.

SLR-MA – 532

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
MATHEMATICS (Old – CGPA)
Functional Analysis (Paper – XI)

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. **1.** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) : **5**

1) An orthonormal set in a Hilbert space is ____________.
2) In a Hilbert space H, an orthonormal set S is complete iff __________.
3) Let N and N' be a normed linear spaces and let T be bounded linear transformation of N into N' then $||T|| =$ _____________.
4) T is continuous if and only if T is ______________.
5) An operator P on Hilbert space H is a projection on H iff ___________.

B) State whether the following statements are **true** or **false** : **4**

1) If $\{e_i\}$ is an othonormal set in a Hilbert space H then

$$\sum |(x, e_i)|^2 \geq \|x\|^2 \; \forall x \in H.$$

2) If S is a nonempty subset of a Hilbert space H then $S^{\perp} = S^{\perp\perp}$.
3) $S^{\perp}$ is the set of all those vectors in H which are orthogonal to every vector in S.
4) On a finite dimensional space all norms are equivalent.

P.T.O.

C) Choose the correct alternative of following (**one** mark **each**) : **5**

1) i) The empty set is convex set.

ii) The set consisting of one point is convex.

a) Only (i) is false b) Only (ii) is false

c) Both (i) and (ii) true d) Both (i) and (ii) are false

2) A subset of separable normed linear space is

a) Separable b) Not separable

c) Connected d) Not connected

3) A normed linear space N is reflexive if

a) N = N* b) N = N** c) $N = N^{\perp}$ d) None of these

4) Let f, $g \in L_p$ where $1 \leq p < \infty$ then

a) $||f + g||_p \leq ||f||_p + ||g||_p$

b) $||f + g||_p \geq ||f||_p + ||g||_p$

c) $||f + g||_p \leq ||f||_p \cdot ||g||_p$

d) None of these

5) If T is an operator on Hilbert space H then (Tx, x) = 0 $\forall x \in H$ iff

a) T = I b) T = 0

c) T is non zero d) None of these

2. a) Show that the adjoint operation is one-one anto as a mapping of B (H) into itself. **4**

b) Let N and N' are normed linear spaces and let T be linear transformation of N into N'. Then prove that T is continuous either at every point of N or at no point of N. **3**

c) If S is nonempty subset of a Hilbert space H then show that $S^{\perp\perp}$ is the closure of the set of all linear combinations of vectors in S. **4**

d) Prove that every complete subspace of a normed linear space is closed. **3**

3. a) Let N be a normed linear space and suppose two norms $|| \ ||_1$ and $|| \ ||_2$ are defined on N. Then prove that these norms are equivalent iff there exist positive real numbers m and M such that $m \, || x ||_1 \leq || x ||_2 \leq M \, || x ||_1, \ \forall x \in N$. **7**

b) Let S be a non empty subset of a Hilbert space H. Then prove that $S^{\perp}$ is a closed linear subspace of H. **7**

4. a) If N_1 and N_2 are normal operators on a Hilbert space H with the property that either commutes with the adjoint of the other then prove that $N_1 + N_2$ and $N_1 N_2$ are also normal operators. **7**

   b) Let M be a closed linear subspace of a normed linear space N and x_0 is a vector not in M. Then prove that there exist a functional F in N* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$. **7**

5. a) Let T be an operator on a normed linear space N then prove that its conjugate T* defined by $T^*: N^* \to N^*$ as $[T^*(f)](x) = f(T(x))$ for all $f \in N^*$ and all $x \in N$ is an operator on N*, also show that $\phi : B(N) \to B(N^*)$ $\phi(T) = T^* \forall \; T \in B(N)$ is an isometric isomorphism of B(N) into B(N*). **8**

   b) If T is normal operator on a Hilbert space H and λ is any scalar then prove that $T - \lambda I$ is also normal. **6**

6. a) If M is proper closed linear subspace of a Hilbert space H then prove that there exist a nonzero vector z_0 in H such that $z_0 \perp M$. **8**

   b) If M is linear subspace of a Hilbert space H then show that M is closed $\Leftrightarrow$ $M = M^{\perp\perp}$. **6**

7. a) Let N and N' are normed linear space and T be continuous linear transformation of N into N'. If M is kernel of T then show that T induces a natural linear transformation T' of $\frac{N}{M}$ into N' and $\|T'\| = \|T\|$. **7**

   b) Let M be a closed proper subspace of a normal linear space N and let a be real number such that $0 < a < 1$. Then prove that there exist a vector $x_a \in N$ such that $\|x_a\| = 1$ and $\| x - x_a \| \geq a$ for all $x \in M$. **7**

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Seat No.	

M.Sc. II (Semester – III) (Old) (CGPA) Examination, 2016
MATHEMATICS
Paper No. – XII : Advanced Discrete Mathematics

Day and Date : Thursday, 31-3-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

***Instructions**: 1) Q. No. **1** and Q. No. **2** are **compulsory**.*
*2) Solve **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate marks.*

1. A) Fill in the blanks : **5**

i) A ring in which every element is __________ is known as Boolean ring.

ii) If edges are directed in a graph then the graph is called ____________

iii) The degree of graph G is always __________

iv) A ___________ graph with n vertices and (n – 1) edges is a Tree.

v) In simple graph _____________ graph contains maximum number of edges.

B) Select correct alternative : **5**

i) If a lower bound of A succeedes every other bound of A then it is called __________

a) Infimum of A b) Maximal of A
c) Greatest lower bound of A d) Both (a) & (c)

ii) A complete graph with 'n' vertices have __________ edges.

a) n(n – 2) b) n(n – 1)
c) $\frac{n(n-1)}{2}$ d) $\frac{n(n-2)}{2}$

iii) A connected graph with 'n' vertices and (n – 1) edges is ____________

a) Tree b) Complete graph
c) Bipartite graph d) Simple graph

iv) An expression for geometric series $1/(1-x)^n$ is ___________

a) $\sum_{r=0}^{\infty} {}^{(n-r)}C_r x_r$

b) $\sum_{r=0}^{\infty} {}^{(n-1+r)}C_r x^r$

c) $\sum_{r=0}^{\infty} {}^{(n+r)}C_r x^r$

d) $\sum_{r=0}^{\infty} {}^{(n+1-r)}C_r x^r$

v) A complemented distributive lattice with 0 & 1 is called _________

a) Boolean Ring

b) Boolean Algebra

c) Modular lattice

d) None of above

C) State **True** or **False** : **4**

i) Every semi modular lattice is modular lattice.

ii) Every complete graph is regular graph.

iii) A pedent vertex is called leaf of tree.

iv) If A & B are infinite set then

$|A \cup B| = |A| + |B| - |A \cap B|$

2. a) Prove that totally ordered set is lattice. **4**

b) Write note on Matrix representation of a graph. **4**

c) How many spanning Trees for K_4 ? Draw all that trees. **3**

d) Find coefficient of x^{27} in $(x^4 + x^5 + x^6 + ...)^5$. **3**

3. a) Show that a partially ordered set with lowest element '0' is complete if every non empty set has l.u.b. **7**

b) Prove that the lattice of all normal subgroup of a group is modular lattice. **7**

4. a) Let a graph G be a non empty graph with at least two vertices then prove that G is bipartite graph iff it has no odd cycle. **8**

b) If G be a connected graph then G is Tree then prove that every edges of G is bridge. **6**

5. a) Let L and L' be any two lattices then show that a bijective function $f : L \rightarrow L'$ is lattice isomorphism iff both f & f' preserve order. **7**

b) Find the number of integer between 1 to 1000 both inclusive which are divisible by 3, 7, 9. **7**

6. a) Solve the recurrence relation

$a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = 1$, $a_1 = 11$. **7**

b) Find the general solution of

$a_r - 5a_{r-1} + 6a_{r-2} = 8r + 5$. **7**

7. a) If <B, +, . > with '0' and '1' is Boolean ring then prove that <B, $\wedge$, $\vee$, 0, 1> is Boolean algebra. **9**

b) Prove that every boolean ring is commutative ring. **5**

SLR-MA – 534

Seat No.	

M.Sc. (Semester – III) (Old) (CGPA) Examination, 2016
MATHEMATICS
Elective – I : Linear Algebra (Paper – XIII)

Day and Date : Saturday, 2-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions *: i) Figures in **right** indicate **full** marks.*
*ii) Q. No. **1** and Q. No. **2** are **compulsory**.*
*iii) Solve **any three** questions from Q. No. **3** to Q. No. **7**.*

1. a) Fill in the blanks :

i) If V is vector space, a ____________ in V is a maximal proper subspace of V.

ii) A form f on V is called non negative if _________

iii) A linear operator T on V is called normal if _______

iv) If λ is an eigenvalue of A, then $|A - \lambda I|$ = _________ **(1+1+1+1)**

b) State whether **true** or **false** :

i) Every characteristic polynomial of A divides its minimal polynomial.

ii) Similar matrices have the same characteristic polynomial.

iii) If minimal polynomial for T is product of linear polynomial then T is diagonalizable.

iv) For a finite dimensional vector space V, dimV = dimV**. **(1+1+1+1)**

c) Define the following :

i) Annihilator of a set

ii) Inner product space

iii) Linear functional. **(2+2+2)**

P.T.O.

2. a) If A is a complex 5 × 5 matrix with characteristic polynomial $f = (x - 2)^3 (x + 7)^2$ and minimal polynomial $(x - 2)^2 (x + 7)$, what is the Jordan form for A ?

   b) Let V be a finite dimensional vector space over the field F. Show that each basis for V* is the dual of some basis for V.

   c) Find a 3 × 3 matrix for which minimal polynomial is x^2.

   d) Show that an orthogonal set of non zero vectors is linearly independent. **(4+4+3+3)**

3. a) Let E_1, ... E_k be k linear operators on V which satisfying conditions :

      i) Each E_i is a projection

      ii) $E_iE_j = 0$ if $i \neq j$

      iii) $I = E_1 + ... E_k$.

      Let W_i be the range of E_i. Then show that $V = W_1 \oplus ...... \oplus W_K$.

   b) Let V be a finite dimensional inner product space. If T and U are linear operators on V and c is a scalar, show that

      i) $(T + U)^* = T^* + U^*$

      ii) $(cT)^* = \bar{c}\, T^*$

      iii) $(TU)^* = U^*T^*$

      iv) $(T^*)^* = T$ **(7+7)**

4. a) Let T be a linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$. Prove that T is not diagnolizable.

   b) Let V be a finite dimensional vector space over the field F and T be a linear operator on V. Show that T is triangulable iff the minimal polynomial for T is a product of linear polynomials over F. **(7+7)**

5. a) Let V be a finite dimensional vector space over the field F. For each vector α in V define $L_\alpha(f) = f(\alpha)$, $f \in V^*$. Show that the mapping $\alpha \to L_\alpha$ is an isomorphism of V onto V**.

   b) Let V be a finite dimensional vector space over the field F, and let $\mathcal{B} = \{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a basis for V. Then show that there is a unique dual basis $\mathcal{B}^* = \{f_1, f_2, ..., f_n\}$ for V* such that $f_i(\alpha_j) = \delta_{ij}$. Also prove that for each linear functional f on V we have $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and for each vector α in V we have $\alpha = \sum_{i=1}^{n} f_i(\alpha)\, \alpha_i$. **(7+7)**

6. a) Let T be a linear operator on an n dimensional vector space V. The characteristic and minimal polynomials for T have the same roots, except for multiplicities.

   b) Let T be a linear operator on the finite dimensional vector space V. Let $c_1, c_2, ..., c_k$ be the distinct characteristic values of T and let W_i be the space of characteristic vectors associated with the characteristic value c_i. If $W = W_1 + W_2 + ... W_k$ then show that $\dim W = \dim W_1 + \dim W_2 + ... + \dim W_k$.

   In fact, show that if $\mathfrak{B}_i$ is an ordered basis for W_i, the $\mathfrak{B} = (\mathfrak{B}_1, \mathfrak{B}_2, ..., \mathfrak{B}_k)$ is an ordered basis for W. **(7+7)**

7. a) Apply the Gram Schmidt process to the vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$, $\beta_3 = (0, 3, 4)$, to obtain an orthonormal basis for R^3 with the standard inner product.

   b) Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W, then

      i) The null space of T^t is equal to the annihilator of the range of T.

      ii) If V and W are finite dimensional then prove that $\text{rank}(T^t) = \text{rank}(T)$. **(7+7)**

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SLR-MA – 535

Seat No.	

M.Sc. – II (Semester – III) (Old) (CGPA) Examination, 2016
MATHEMATICS
Elective – II : Modeling and Simulation (Paper – XIV)

Day and Date : Tuesday, 5-4-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *i) Question no. **1** and **2** are **compulsory**.*
*ii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iii) Figures to the **right** indicate **full** marks.*
*iv) **Use** of simple or scientific calculator is **allowed**.*

1. A) Select most correct alternative : **10**

i) Customers after joining the queue, wait for some time and leave the service systems due to intolerable delay, so they

a) renege b) balk

c) jockey d) (a) or (c)

ii) In M/M/1:∞/FCFS Queue model if λ is mean customer arrival rate and μ is mean service rate then the probability of server being busy is equal to

a) $\frac{\lambda}{\mu-\lambda}$ b) $\frac{\mu}{\mu-\lambda}$ c) $\frac{\lambda}{\mu}$ d) $\frac{\mu}{\lambda}$

iii) A manufacturer has to supply his customers 600 units of his product per year. Shortages are not allowed and the storage (carrying) cost amounts to Rs. 0.60 per unit per year. The set up cost (ordering) per run is Rs. 80. The optimal order quantity is

a) 160000 b) 450 c) 200 d) 400

iv) PERT is used when there is a good deal of ___________ regarding the time taken by various activities in the project.

a) certainty b) uncertainty

c) both (a) and (b) d) none of these

v) What will be the corresponding random observation generated on continuous uniform distribution over (– 5, 5) when a random number generated between 0 and 1 is 0.7352 ?

a) 12.352 b) 5.7352

c) – 2.352 d) 2.352

vi) If a r.v. X follows standard normal distribution then the variance of X is

a) – 1 b) 0

c) 1 d) None of these

vii) Repetition of n independent Bernoulli trial reduces to

a) Poisson distribution

b) Binomial distribution

c) Hypergeometric distribution

d) Geometric distribution

viii) Economic Order Quantity (EOQ) results in

a) Equalisation of carrying cost and procurement (ordering) cost

b) Minimization of set up cost

c) Favourable procurement price

d) Reduced chances of stock outs

ix) The process of simulation

a) is a powerful mathematical technique

b) is often referred to as "Monte-Carlo" simulation

c) usually require use of computers to solve the problems

d) involve the criterion wherein the output of simulation model is independent of the simulation run

x) In critical path analysis, the word CPM mean

a) Critical Path Method

b) Crash Project Management

c) Critical Project Management

d) Critical Path Management

B) Fill in the blanks : **4**

i) If the exponential distribution is given as $f(x) = 2e^{-2x}$, $0 \leq x \leq \infty$.then the mean of the distribution is ______________.

ii) The long form of PERT is ______________.

iii) Simulation of systems in which the state changes smoothly or continuously with time are called _____________ systems.

iv) In queue model completely specified in the symbolic form (a/b/c):(d/e), the last symbol e specifies _____________.

2. A) i) Define Poisson distribution and state its mean and variance. **4**

ii) An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1800 per year. What should be the economic order quantity per order, if the cost of placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise ? **4**

B) i) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait ? **3**

ii) Define a Markov Chain. **3**

3. A) Describe the deterministic inventory model of EOQ with uniform demand and no shortages. **7**

B) A project schedule has the following activities and the time (in months) of completion of each activity is as follows :

Activity	1-2	1-3	2-4	3-5	4-5
Time	8	10	5	6	4

Draw the network diagram and find the minimum time of completion of the project, slack times for each activity and critical path. **7**

4. A) Give the rules for constructing the network diagram in network analysis. **7**

B) ABC Bakery keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here :

Daily Demand	0	15	30	45	60	75
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Consider the following sequence of random numbers :

0.45, 0.70, 0.29, 0.58, 0.66, 0.17, 0.15, 0.34, 0.88, 0.14.

Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data. **7**

5. A) Explain briefly the important characteristics of queueing system. **7**

B) Write an algorithm of generating m random observations from binomial distribution with parameters n and p. **7**

6. A) What are the advantages and limitations of using simulation ? **7**

B) Give he steps of Monte-Carlo simulation technique. **7**

7. A) Differentiate between PERT and CPM. **7**

B) Explain generation of a random sample from normal distribution. **7**

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SLR-MA – 536

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
MATHEMATICS (Old) (CGPA)
Elective – III Numerical Analysis
Paper – XV

Day and Date : Thursday, 7-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q.**1** and Q.**2** are **compulsory**.*
*2) Attempt any three questions from Q.No. **3** to Q. No. **7.***
*3) Figures to the **right** indicate **full** marks.*
*4) Use of calculator is **allowed.***

1. A) Fill in the blanks (**one** mark **each**).

i) The averaging operator μ is defined as ________

ii) The error in Simpson's $1/3^{rd}$ rule is given by __________

iii) In Gaussian elimination we reduce the coefficient matrix to _________

iv) Newton-Raphson method fails when __________

B) Choose correct alternative (**one** mark **each**)

i) The real root of the equation $x^3 - \cos x = 0$ lies between.

a) 0 and 1 b) 1 and 2 c) – 1 and 0 d) none

ii) Newtons forward interpolation formula used for interpolation.

a) Near the beginning of set of tabular values

b) End of set of tabular values

c) For both

d) None

iii) Using Householder's method the matrix is reduced to

a) upper triangular b) lower triangular

c) symmetric tridiagonal form d) none

P.T.O.

iv) If $f(x) = x^3$ then the value of the first divided difference of the argument 1 and 2 is

a) 5 b) 6 c) 7 d) 8

v) The relation between E and Δ is given by

a) $\Delta = E - 1$ b) $\Delta = E + 1$ c) $\Delta = E$ d) none

C) Define the following (**one** mark **each**).

i) Round off error

ii) Relative error

iii) Percentage error

iv) Rate of convergence

v) Inherent error.

2\. i) State Trapezoidal, Simpsons $1/3^{rd}$ and $3/8^{th}$ formula. **3**

ii) Derive error in series approximation. **4**

iii) Using the method of separation of symbols show that

$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \ldots (-1)^n u_{x-n}$$ **4**

iv) Explain partial pivoting. **3**

3\. i) Use the method of false position to find a real root of the equation $x - e^{-x} = 0$. **7**

ii) Find a cubic polynomial which agrees on following set of tabular values. **7**

X	0	1	3	4
y = f(x)	– 12	0	12	24

by using Lagrange's method.

4\. i) Decompose the matrix **7**

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

in to the form LU.

ii) Perform four iterations of the Newton-Raphson method to obtain a real root of the equation $x^4 - 11x + 8 = 0$ **7**

5. i) Derive rate of convergence of secant method. **8**

ii) Find y (4) using Newton's backward difference interpolation formula from following data **6**

X	0	1	2	15
y=f(x)	2	3	12	3587

6. i) Find the largest eigen value and corresponding eigen vector of the matrix **7**

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

ii) Use Gauss elimination method to solve **7**

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

7. i) Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's a/3rd rule with $h = \frac{\pi}{12}$. **7**

ii) Show that the differential equation $\frac{d^2y}{dx^2} = -xy$, $y(0) = 1$, $y'(0) = 0$ has the series solution **7**

$$y = 1 - \frac{x^3}{3!} + \frac{1 \times 4}{6!}x^6 - \frac{1 \times 4 \times 7}{9!}x^9 + \ldots\ldots$$

Seat No.	

M.Sc. – II (Semester – IV) (New) (CGPA) Examination, 2016
MATHEMATICS
Measure and Integration (Paper XVI)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1\. A) Fill in the blanks (**one** mark **each**) : **5**

1) For a signed measure γ defined on a measurable space $\langle x, \mathcal{B} \rangle$. Then for $E \in \mathcal{B}$, total variation $|\gamma|(E) =$ ______________

2) Each measure space can be completed by the addition of subsets of sets of ______________

3) A set that is both positive and negative w.r.t. signed measure γ is called a ______________ set.

4) If $\{f_n\}$ is a sequence of nonnegative measurable functions which converges to f almost everywhere and if $f_n \leq f$ for all n then $\int f =$ ______________

5) Let μ be a measure on an algebra $\mathcal{A}$, the outer measure induced by μ is defined as $\mu^*(E) =$ ______________

B) State **true** or **false** (**one** mark **each**) : **5**

1) If γ_1 and γ_2 are mutually singular w.r.t. μ then $\gamma_1 + \gamma_2$ is not mutually singular w.r.t. μ.

2) Collection of locally measurable sets is a σ-algebra.

3) If E and F be disjoint sets, then $\mu_*(E \cup F) \leq \mu_*(E) + \mu_*(F)$.

4) Union of two positive sets is positive.

5) Every subset of a positive set is itself positive.

C) Choose the correct answer (**one** mark **each**) : **4**

1) Consider the two statements.

I) If γ is a measure $\Rightarrow \gamma$ is a signed measure.

II) If γ is a signed measure $\Rightarrow \gamma$ is a measure.

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

2) A measure μ on a measurable space is a saturated if

a) Every sebset of X is a measurable

b) Every locally measurable subset of X is a measurable

c) Every measurable subset of X is a locally measurable

d) None of these

3) The set E is said to be measurable w.r.t. μ^* if for every set A.

a) $\mu^*(E) = \mu^*(A \cap E) + \mu^*(A \cap E')$

b) $\mu^*(E) = \mu^*(E \cap A) + \mu^*(E \cap A')$

c) $\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap E')$

d) $\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap A')$

4) In Fubini's theorem, measures μ and γ are

a) Finite b) Complete

c) σ-finite d) Semi finite

2. a) Show that every σ- finite measure is a saturated.

b) Show that if E is any measurable set, then $-\gamma^-(E) \le \gamma(E) \le \gamma^+(E)$ and $|\gamma(E)| \le |\gamma|(E)$.

c) Prove that if $\mu^*(E) < \infty$, then E is a measurable if and only if $\mu_*(E) = \mu^*(E)$.

d) Let x be a point of X and E is a set in $\mathcal{R}_{\sigma\delta}$. Then prove that E_x is a measurable subset of Y. **(3+4+4+3)**

3. a) Define a complete measure space. Let < X, $\mathcal{B}$, μ > be a measure space. Show that there exists a complete measure space < X, $\mathcal{B}$ μ_o> such that

   i) $\mathcal{B} \subseteq \mathcal{B}_o$

   ii) $E \in \mathcal{B}_o \Leftrightarrow E = A \cup B$, where $B \in \mathcal{B}$ and $A \subseteq C$, $C \in \mathcal{B}$ with $\mu(C) = 0$.

   iii) If $E \in \mathcal{B} \Rightarrow \mu(E) = \mu_o(E)$.

   b) State and prove Monotone convergence theorem. **(8+6)**

4. a) Let g be integrable over E, and suppose that $\{f_n\}_{n=1}^{\infty}$ is a sequence of measurable functions such that, on E, $|f_n(x)| \leq g(x)$ and almost everywhere on E, $f_n(x) \rightarrow f(x)$. Then show that $\int_E f = \lim_{n \to \infty} \int f_n$.

   b) State and prove Hahn decomposition theorem. **(8+6)**

5. a) State Radon-Nikodym theorem. Show that the condition of σ – finiteness is essential in the theorem.

   b) State and prove Jordan decomposition theorem. **(6+8)**

6. a) Let $\{A_i\}_{i=1}^{\infty}$ is a disjoint sequence of sets in $\mathcal{A}$ then show that

$$\mu_* \left[E \cap \left(\bigcup_{i=1}^{\infty} A_i \right) \right] = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$$

   b) If $A \in \mathcal{A}$ then show that $\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap E')$ for any $E \subseteq X$; where μ_* is an inner measure and μ^* is an outer measure. **(8+6)**

7. a) State and prove Tonelli's theorem.

   b) Define an inner measure of a set. Show that for any set A, $\mu_*(A) \leq \mu^*(A)$. Equality holds if $A \in \mathcal{A}$, where $\mathcal{A}$ is an algebra of sets. **(8+6)**

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SLR-MA – 538

Seat No.	

M.Sc. II (Semester – IV) (New) (CGPA) Examination, 2016
MATHEMATICS (Paper – XVII)
Partial Differential Equations

Day and Date : Friday, 1-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

***Instructions** : 1) Question no. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q.No. **3** to question no. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative (**one** mark **each**).

i) If there is a functional relation between two functions u(x, y) and v(x, y) not involving x and y explicitly then

a) $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial v}{\partial y} \neq 0$ b) $\frac{\partial (u, v)}{\partial (x, y)} = 0$

c) $\frac{\partial v}{\partial x} = 0$ and $\frac{\partial u}{\partial y} \neq 0$ d) $\frac{\partial (u, v)}{\partial (x, y)} \neq 0$

ii) A p.d.e. $(n - 1)^2 u_{xx} - y^{2n} u_{yy} = n \cdot y^{2n-1} u_y$ where n is an integer is of hyperbolic type if

a) $n = 1$ b) $n < 1$ c) $n > 1$ d) $n = 0$

iii) The two solutions of Neumann problem differ by

a) function of x b) function of y

c) function of x and y d) constant

iv) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

a) $z = f_1(y + x) + f_2(y - x)$ b) $z = f_1(y + x) + f_2(y - x)$

c) $z = f_1(y + x) + xf_2 (y + x)$ d) $z = f(x^2 - y^2)$

B) Fill in the blank (**one** mark **each**).

i) Solution of ptanx + qtany = tanz is __________

ii) The differential equation of the set of all right circular cones whose axis coincide with z-axis is _________

iii) The equations f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 are compatible then they have __________ parameter family of common solution.

iv) The Pfaffian differential equation for continuous function is _________

v) For any constant a, parametric equation x = asinucosv, y = asinusinv, z = acosu represents __________

C) State **true** or **false** (**one** mark **each**).

i) When the number of arbitrary constants is less than the number of independent variables then the elimination of arbitrary constants gives more than one pole of order one.

ii) There does not exists an integrating factor for a Pfaffian differential equation in two variables.

iii) The solution of Dirichlet problem if it exists need not be unique.

iv) Order of ptany + qtanx = $\sec^2 z$ is two.

v) A function f(x, y) is said to be homogeneous of degree n if it satisfies $f(\lambda x, \lambda y) = \lambda^n f(x, y)$.

2. a) Show that the surface f(x, y, z) = $x^2 + y^2 + z^2 = c$, c > 0 can form an equipotential family of surfaces. **3**

b) Solve $x^2 p + y^2 q = nxy$. **4**

c) Find the integral surface of yp + xq = z – 1 which passes through the curve $z = x^2 + y^2 + z$, y = 2x. **3**

d) Determine the regions where the equation $u_{xx} - 2x^2 u_{xz} + u_{yy} + u_{zz} = 0$ is of hyperbolic elliptic or parabolic type. **4**

3. a) Prove that necessary and sufficient condition for integrability of $dz = \phi(x, y, z)\,dx + \psi(xyz)\,dy$ is [f, g] = 0. **7**

b) Find the integral surface of the equation pq = z passing through c : $x_0 = 0$, $y_0 = s$, $z_0 = s^2$. **7**

4. a) Prove that necessary and sufficient condition that the Pfaffian differential equation $\bar{x} \cdot d\bar{r} = P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$ be integrable is that $\bar{x} \cdot \text{curl}\,\bar{x} = 0$. **7**

b) Show that the equation z = px + qy is compatible with any equation f(x, y, z, p, q) = 0 which is homogeneous in x, y, z. **7**

5. a) Find the integral surface of $zz_x + z_y = 0$ containing the initial data curve

C : $x_0 = s$, $y_0 = 0$, $z_0 = f(s)$ where

$$f(s) = \begin{cases} 1 & s \le 0 \\ 1 - s & 0 \le s \le 1 \\ 0 & s \ge 1 \end{cases}.$$ **7**

b) State and prove Harnack's theorem. **7**

6. a) Find the D'Alembert's solution which describes the vibrations of an infinite string. **7**

b) Solve $p = (z + qy)^2$ by Jacobi's method. **7**

7. a) Show that the solution of

$u_{tt} - c^2 u_{xx} = f(x, t),\ 0 < x < l,\ t > 0$

$u(x, 0) = f(x),\ 0 \le x \le l$

$u_t(x, 0) = g(x),\ 0 \le x \le l$

$u(0, t) = u(l, t) = 0,\ t \ge 0$

if it exists is unique. **7**

b) Find the general integral of $(x^2 + y^2)\,p + 2xyq = (x + y)z$. **7**

Seat No.	

M.Sc. – II (Semester – IV) Examination, 2016
MATHEMATICS (New – CGPA)
Elective – I : Integral Equations (Paper – XVIII)

Day and Date : Monday, 4-4-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Figures to the **right** indicate **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks : **10**

1) The eigen functions corresponding to distinct eigen values of symmetric kernel are ________________

2) The Bessel's inequality is given by ______________________

3) The homogeneous Volterra integral equation of second kind is of the form __________________

4) If k(x, t) is real symmetric, continuous and k(x, t) $\neq$ 0 then all the characteristic constants are ______________

5) The multiplicity of any non-zero eigen value is finite for every symmetric kernel for which ___________

6) The eigen values of symmetric L_2-kernel form a finite or infinite sequence $\{\lambda_n\}$ with _____________ limit point.

7) If k(x, t) = 10 (x – t) then it is of _____________ type kernel.

8) If F(x, t) and $\frac{\partial F}{\partial x}$ be continuous functions of both x and t & let the first derivatives of G(x) and H(x) be continuous. Then the Lebnitz rule is given by ________________

9) Every symmetric kernel with a norm not equal to zero has _________ eigen value.

10) ________________ process is used to construct orthonormal vectors.

B) **True** or **false** : **4**

1) The m^{th} iterated kernel $k_m(x, t)$ satisfies the relation

$$k_m(x, t) = \int_a^b k_p(x, z) k_{m-p}(z, t) dz$$; where p is any integer less than m.

2) The Laplace transform can be used to solve the integral equation if the kernel is of degenerate type.

3) The set of eigen values of second iterated kernel coincides with the set of eigen values of Given kernel.

4) The sequence of eigen functions of a symmetric kernel can be made orthonormal.

2. a) Show that the function y(x) = 1 – x is the solution of the integral equation $\int_0^x e^{x-t} y(t) dt = x$.

b) State Hilbert Schmidt theorem.

c) Convert $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with y(0) = 1 and $y'(0) = -2$ into Volterra equation.

d) Let $R(x, t; \lambda)$ be the reciprocal kernel of the Volterra integral equation

$$y(x) = f(x) + \lambda \int_a^x k(x, t) y(t) dt$$. Then prove that the reciprocal kernel satisfy the following equation $R(x, t; \lambda) = k(x, t) + \lambda \int_t^x k(x, z) R(z, t; \lambda) dz$ **(4+3+4+3)**

3. a) Construct Green's function for the homogeneous boundry value problem $\frac{d^4y}{dx^4} = 0$ with y(0) = $y'(0)$ = y(1) = $y'(1)$ = 0.

b) Convert $y''(x) + \lambda y(x) = x$ with the conditions y(0) = 0 and $y(\pi) = 0$ into an integral equation. **(9+5)**

4. a) Explain successive approximation method.

b) Find the eigen values and eigen functions of the equation

$y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + t) y(t) dt$. where λ is not an eigen value. **(7+7)**

5. a) Show that the function y(x) = cos 2x is a solution of the integral equation

$$y(x) = \cos x + 3\int_0^{\pi} k(x, t)y(t)dt \text{ where } k(x, t) = \begin{cases} \sin x \cos t & ; 0 \le x \le t \\ \cos x \sin t & ; t \le x \le \pi \end{cases}.$$

b) Solve the Abel's integral equation $\int_0^t \frac{Y(x)}{\sqrt{t-x}} dx = 1 + t + t^2$. **(7+7)**

6. a) Reduce the following boundry value problem $y''(x) + xy(x) = 1$ with $y(0) = y(1) = 0$

to the integral equation $y(x) = \int_0^1 G(x, t)y(t)dt - \frac{1}{2}x(1-x)$ where

$$G(x,t) = \begin{cases} x(1-t) & ; x < t \\ t(1-x) & ; t > x \end{cases}.$$

b) Prove that if a kernel is symmetric then all its iterated kernels are also symmetric. **(7+7)**

7. a) Find the resolvent kernel of the following kernel $k(x, t) = e^{x^2 - t^2}$.

b) Solve the following symmetric integral equation using Hilbert Schmidt theorem

$$y(x) = 1 + \lambda\int_0^{\pi} \cos(x+t)y(t)dt.$$ **(7+7)**

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SLR-MA – 540

Seat No.	

M.Sc. – II (Semester – IV) Examination, 2016
MATHEMATICS (Paper – XIX)
(Elective – II) Operations Research (New) (CGPA)

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q.* ***3*** *to* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternatives of the following questions.

i) Which of the following is not associated with an LPP ?

a) Additivity b) Uncertainty
c) Proportionality d) Divisibility

ii) What is the optimum BFS of following LPP ?

Max $Z = x_1 + x_2$

Subject to, $x_1 + 2x_2 \leq 4$, $3x_1 + 2x_2 \leq 10$, $x_1 \geq 0$, $x_2 \geq 0$

a) $x_1 = 0, x_2 = 2$
b) $x_1 = 2, x_2 = 1$
c) $x_1 = \frac{10}{3}, x_2 = 0$
d) $x_1 = 2, x_2 = 2$

iii) A two person game is said to be zero sum game, if

a) Gain of one player is exactly matched by a loss to the other so that their sum is equal to zero
b) Both players must have exact number of strategies
c) Diagonal entries of pay-off matrix are zero
d) Gain of one player does not match to the loss of other player

P.T.O.

iv) Given a system of m simultaneous linear equations with n unknowns ($m < n$). The number of basic variables will be

a) n b) m

c) n – m d) None of the above

v) If $X'QX$ is positive semi definite then, it is

a) Strictly convex b) Strictly concave

c) Convex d) Concave

B) Fill in the blanks.

i) Basic feasible solution to the linear programming problem corresponds to ____________ point.

ii) If the players select the same strategy each time, then it is referred as ____________

iii) Quadratic programming problem is the particular case of ____________ programming problem.

iv) In dual simplex method, we starts with ____________ solution and go to the optimum basic feasible solution.

v) Branch and bound method use to solve ____________ programming problem.

C) State whether following statements are **true** or **false**.

i) Quadratic programming problem have quadratic objective function as well as quadratic constraint.

ii) Primal problem have unbounded solution, then dual also possess unbounded solution.

iii) Every pay-off matrix has saddle point.

iv) A necessary and sufficient condition for a BFS to be an optimum is $z_j - c_j \geq 0$. **(5+5+4)**

2. a) Explain graphical method to solve LPP with two variables.

b) Define the terms :

i) Convex set ii) Convex combinations

iii) Basic feasible solution iv) Unbounded solution.

c) Show that set of all feasible solutions is convex set.

d) Show that dual of dual is primal. **(4+4+3+3)**

3. a) Describe two phase simplex method.

   b) Solve the following LPP using simplex method.

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$ Subject to the constraint

   $x_1 + 2x_2 + x_2 \leq 430$, $3x_1 + 2x_2 <= 460$, $x_1 + 4x_3 <= 420$, $x_1, x_2, x_3 \geq 0$. **(7+7)**

4. a) Explain Branch and Bound method to solve integer linear programming.

   b) Use Gomory's cutting plane method and solve the following IPP

   Max $Z = x_1 + 2x_2$ Subject to the constraints :

   $2x_2 \leq 7$, $x_1 + x_2 \leq 7$, $2x_1 \leq 11$, x_1, x_2 are non-negative integers. **(7+7)**

5. a) What is mean by duality in linear programming problem ? Also state and prove weak law of duality.

   b) State and prove complementary slackness theorem. **(7+7)**

6. a) Solve the following QPP by Wolfe's method.

   Max $z = 2x_1 + x_2 - x_1^2$ Subject to the constraint

   $2x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$, $x_1, x_2 \geq 0$.

   b) Obtain the Kuhn Tucker condition of optimality for a quadratic programming problem. **(7+7)**

7. a) For two person zero sum game show that maximin value of the game is less than or equal to the minimax value of the game.

   b) Solve the game with the following payoff matrix.

Player B

$$\text{Player A}\begin{bmatrix} 3 & -1 & -3 \\ -2 & 4 & 1 \\ -5 & -6 & 2 \end{bmatrix}$$

(7+7)

SLR-MA – 541

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
MATHEMATICS (Paper – XX)
Elective – III : Probability Theory (New) (CGPA)

Day and Date : Saturday, 9-4-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. **(1)** and Q.No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q.No. **(3)** to Q.No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative

1) α : Union of fields is a field

β : Intersection of fields is a field

a) only α is true b) only β is true

c) both α and β are true d) both α and β are false

2) Convergence in probability implies ____

a) Almost sure convergence b) Convergence in distribution

c) Convergence in r^{th} mean d) None of these

3) Let P (·) is a probability measure defined on (Ω , $\mathbf{F}$). Then p (Ω) = 1 is called _____ property of measure.

a) finite additivity b) σ-additivity

c) normed d) non-negativity

4) Characteristic function $\phi(t)$ of random variable X is _____

a) discrete b) continuous

c) may be (a) or (b) d) cannot be determined

P.T.O.

5) Let $\{A_n\}$ be a sequence of sets. Then _____

a) lim A_n always exist

b) $\underline{\lim}\ A_n$ and $\overline{\lim}\ A_n$ always exist

c) $\overline{\lim}\ A_n \subseteq \underline{\lim}\ A_n$

d) none of these

B) Fill in the blanks :

1) Characteristic function of Binomial random variable is ______

2) Lebesgue measure of any singleton set is ______.

3) Every field must contain _______ and ______.

4) If ϕ is characteristic function of X then $\bar{\phi}$ is characteristic function of _____

5) Minimal σ - field induced by the indicator function I_A is _____.

C) State whether following statements are **true** or **false** :

1) Convergence in r^{th} mean is stronger than convergence in probability.

2) E (X + Y) = E (X) + E (Y), provided only if X and Y are independent.

3) If $\phi(t)$ is characteristic function then $|\phi(t)|^2$ is also a characteristic function.

4) Counting measure is a finite measure.

2. a) Explain the terms :

i) Pairwise independence ii) Mutual independence.

b) Write short note on the following :

i) Lebesgue-Stieljes measure.

ii) Strong law of large numbers. **(6+8)**

3. a) Define field and σ - field of subsets of a set Ω . Show that σ - field is a field. Is the converse true ? Justify.

b) Consider the function defined by

$$X(\omega) = \begin{cases} C_1, & \text{if } \omega \in A_1 \\ C_2, & \text{if } \omega \in A_2 \\ C_3, & \text{if } \omega \in A_3 \end{cases}.$$

Where C_1, C_2 and C_3 are distinct. Obtain minimal σ - field induced by X. **(7+7)**

4. a) Define measurable function. Prove that an indicator function defined on $(\Omega, \mathbb{F})$ is $\mathbb{F}$- measurable if $A \in \mathbb{F}$.

b) Define random variable. Prove that function of random variable is also a random variable. **(7+7)**

5. a) Prove that X is integrable if and only if |X| is integrable.

b) If X and Y are any two random variables then prove that $E(X \pm Y) = E(X) \pm E(Y)$. **(7+7)**

6. a) State the following :

i) Lindeberg – Levy CLT.

ii) Liapounov CLT.

iii) Lindeberg-Feller CLT.

b) State inversion formula. Obtain the distribution corresponding to the characteristic function.

$\phi(t) = e^{-t^2/2}, -\infty < t < \infty$. **(7+7)**

7. a) Define :

i) Convergence in probability.

ii) Convergence almost surely.

iii) Convergence in r^{th} mean.

b) State and prove necessary and sufficient condition for convergence in probability. **(7+7)**

SLR-MA – 542

Seat No.	

M.Sc. – II (Semester – IV) (Old CGPA) Examination, 2016
MATHEMATICS (Paper – XVI)
Measure and Integration

Day and Date : Wednesday, 30-3-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

N. B. : *1) Q. No.* ***1)*** *and Q. No.* ***2)*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*
3) Figures to ***right*** *indicate* ***full*** *marks.*

1. A) State **True** or **False** (**one** mark **each**) :
   1) Every signed measure is measure.
   2) We cannot remove the condition of integrability from the statement of Tonelli's theorem.
   3) Every finite measure space is σ-finite measure space.
   4) Every subset of negative set is negative.
   5) Empty set is null set with respect to any signed measure γ.
   6) For any set $X(\neq, \phi)$, $\mathcal{S} = \{\phi, X\}$ is the smallest σ-algebra.
   7) Lebesgue measure on $\mathbb{R}$ is a finite measure.

   B) Fill in the blanks (**one** mark **each**) :
   1) Set E is negative set with respect to a singed measure γ if ___________
   2) By a total variation of a signed measure γ we means ___________
   3) Let μ and γ be measures on measurable space $\langle X, \mathcal{S} \rangle$ then $u - \alpha\gamma$ is singed measure for ___________ real number.
   4) In Tonelli's theorem the measure μ and γ are ___________
   5) If μ_* is inner measure and μ^* is outer measure then relation between μ_* and μ^* is ___________
   6) If γ and μ are measure on $\mathcal{S}$ both are σ-finite and $\gamma << \mu$ and $\mu << \gamma$ then there Radon Nikodiym derivative is ___________
   7) γ^+ and γ^- are positive and negative variation of γ respectively then $\gamma =$ __________

P.T.O.

2. a) If $\gamma_1 << \mu$ and $\gamma_2 << \mu$ then prove that $C_1\gamma_1 + C_2\gamma_2 << \mu$ where γ_1, γ_2 and μ are measures on $\langle X I I \mathcal{B} \rangle$. **4**
   b) State Fubinis theorem. **3**
   c) Show that countable union of +ve set is +ve. **4**
   d) Define complete measure space and give suitable example. **3**

3. a) Suppose that each α in a dense set D of real numbers there is assigned a set $B\alpha \in \mathcal{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then there is unique measurable extended real valued function f on X such that $F \leq \alpha$ on B_α and $F \geq \alpha$ on $X - B_\alpha$. **8**
   b) Show that $\mu(E_1 \Delta E_2) = 0$ implies $\mu(E_1) = \mu(E_1)$ provided E_1 and $E_2 \in \mathcal{B}$. **6**

4. a) State and prove Jordan Decomposition theorem. **8**
   b) Show by an example that Hann-Decomposition need not be unique. **6**

5. a) Let E and F be disjoint sets then prove that $\mu_*(E) + \mu_*(F) \leq \mu_*(E \cup F) \leq \mu_*(E) + \mu^*(F) \leq \mu^*(E \cup F) \leq \mu^*(E) + \mu^*(F)$. **7**
   b) Let $(X, I\mathcal{B}, \mu)$ be a measure space if $E_i \in \mathcal{B}$, $\mu(E_1) < \infty$ and $E_i \supseteq E_{i+1}$ then, prove that, $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu(E_i)$. **7**

6. a) Let $\langle X, \mathcal{B}, \mu \rangle$ be a measure space. Let E be a measurable set such that $0 < \gamma(E) < \infty$. Show that there exist a positive set A contained in E with $\gamma(A) > 0$. **8**
   b) If $\gamma_1 << \mu$ and $\gamma_2 << \mu$ and μ is σ-finite measure then prove that $\left[\frac{d(\gamma_1 + \gamma_2)}{d\mu}\right] = \left[\frac{d\gamma_1}{d\mu}\right] + \left[\frac{d\gamma_2}{d\mu}\right]$ almost everywhere with respect to μ. **6**

7. a) Show that collection of all Locally-measurable sets is a σ-algebra. **7**
   b) Let X = [0, 1] and $\mathcal{B} = \mathbb{P}(X)$ and μ is counting measure then,
      1) Prove that $\langle X, \mathcal{B}, \mu \rangle$ is measure space.
      2) Prove or disprove $\langle X, \mathcal{B}, \mu \rangle$ is σ-finite measure space. **7**

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Seat No.	

M.Sc. – II (Semester – IV) Examination, 2016
MATHEMATICS (Paper – XVII) (Old) (CGPA)
Partial Differential Equations

Day and Date : Friday, 1-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : 1) *Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) *Attempt* ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) *Figures to the* ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative : **6**

i) The equation $P_p + Q_q = R$ is known as

a) Charpits equation b) Lagranges equation
c) Bernoullis equation d) Clairaut's equation

ii) Eliminating a, b from z = (x + a) (y + b) gives the p.d.e.

a) pq = z b) p/q = z c) p + q = z d) none of these

iii) Equation ptany + qtanx = $\sec^2 z$ is of order

a) 1 b) 2 c) 0 d) none of these

iv) A second order p.d.e. is classified into

a) parabolic b) hyperbolic
c) elliptic d) all of above

v) The equations f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 are compatible then

a) $\frac{\partial(f, g)}{\partial(p, q)} = 0$ and dz = pdx + qdy is integrable

b) $\frac{\partial(f, g)}{\partial(p, q)} \neq 0$ and dz = pdx + qdy is not integrable

c) $\frac{\partial(f, g)}{\partial(p, q)} \neq 0$ and dz = pdx + qdy is integrable

d) None of (a), (b) and (c)

vi) A p.d.e. $(n-1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} u_y$.
where n is an integer is of hyperbolic type if

a) $n = 1$ b) $n < 1$ c) $n > 1$ d) $n = 0$

B) Fill in the blanks : **6**

i) The corresponding p.d.e. for $z = x + ax^2y^2 + b$ is ____________

ii) The complete integral of the p.d.e. $z = px + qy + \log pq$ is ______________

iii) For any constant a, parametric equation

$x = a \sin u \cos v$ $y = a \sin u \sin v$ $z = a \cos u$ represents ______________

iv) The pfaffian differential equation $P(x, y, z)\, dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$ is integrable iff _______________

v) A second order p.d.e. is called parabolic if _____________

vi) The Laplace equation in three dimension is ______________

C) State **true** or **false** : **2**

i) If $u \in c^2$ in $R \times R$ then $u = f(x + 2t)$ is a solution of the second order p.d.e. $u_{xx} - u_{tt} = 0$.

ii) Eliminating arbitrary function f from $z = f(x^2 + y^2)$ we get first order non-linear p.d.e.

2. a) Eliminate the arbitrary function F from $F(x + y, x - Jz) = 0$ and find the corresponding p.d.e. **3**

b) Find the general integral of $y^2p - xyq = x(z - 2y)$. **3**

c) Prove that the solution of Neumann problem is either unique or it differs from one another by a constant only. **4**

d) If $\bar{X} \cdot \text{curl}\, \bar{X} = 0$ where $X = P\bar{i} + Q\bar{j} + R\bar{k}$ and μ is an arbitrary differential function of x, y and z then prove that $\mu\bar{X} \cdot \text{curl}\left(\mu\bar{X}\right) = 0$. **4**

3. a) Reduce the equation $(n-1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} u_y$ where n is an integer, to a canonical form. **7**

b) Find a complete integral of the equation $p^2x + q^2y = z$ by Jacobi's method. **7**

4. a) Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

$0 \le x \le a, 0 \le y \le b$ subject to the boundary conditions $u_x(0, y) = u_x(a, y) = 0$ $u_y(x, 0) = 0$, $u_y(x, b) = f(x)$. **7**

b) Find complete integral of $2(z + xp + yq) = yp^2$ by Charpit's method. **7**

5. a) Explain analytic expression for the Monge Cone at (x_0, y_0, z_0). **7**

b) Show that the equations $f = xp - yq - x$, $g = x^2p + q - xz$ are compatible and find one parameter family of common solutions. **7**

6. a) Prove that a necessary and sufficient condition that the pfaffian differential equation $\overline{X} \cdot d\bar{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ be integrable is that $\overline{X} \cdot \text{curl}\, \overline{X} = 0$. **7**

b) Find particular solution of

$f(x, y, z, p, q) = z - px - qy - p^2 - q^2$. **7**

7. a) Find the integral surface of the p.d.e. $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ through the curve $xz = a^2$, $y = 0$. **7**

b) Suppose that $u(x, y)$ is harmonic in a bounded domain D and is continuous on $\overline{D} = D \cup B$. Then u attains its minimum on the boundary B of D. **7**

SLR-MA – 544

Seat No.	

M.Sc. (Semester – IV) (Old) (CGPA) Examination, 2016
MATHEMATICS
Integral Equations (Paper – XVIII) (Elective – I)

Day and Date : Monday, 4-4-2016 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *i) Figures in **right** indicate **full** marks.*
*ii) Q. No. **1** and Q. No. **2** are **compulsory**.*
*iii) Solve **any three** questions from Q. No. **3** to Q. No. **7***

1. a) Fill in the blanks :

1) A kernel k(x, t) is said to be Hilbert - Schmidt kernel if it is Hermintian and __________

2) A function f(x) is said to be _________ if || f(x)|| = 1.

3) The eigen values of a symmetric kernel form a finite or infinite sequence $\langle \lambda_n \rangle$ with no finite ________

4) The multiplicity of any nonzero eigen value is finite for every symmetric kernel for which $\int_a^b \int_a^b |k(x,t)|^2 \, ds dt$ is _________ **(1+1+1+1)**

b) State whether **true** or **false** :

1) Every symmetric kernel with a norm, not equal to zero has atleast one eigen value.

2) All iterated kernels of symmetric kernel are real.

3) An integral equation is an equation in which known function appears under one or more integral signs.

4) The set of eigen values of the n^{th} iterated kernel coincide with the set of n^{th} root of the eigen values of the given kernel. **(1+1+1+1)**

P.T.O.

c) Define the following :
   1) Linear integral equation.
   2) Convolution type kernel.
   3) Square integrable function. **(2+2+2)**

2. a) Show that the function $u(x) = e^x\left(2x - \frac{2}{3}\right)$ is a solution of the integral equation

$$u(x) + 2\int_0^1 e^{x-t}u(t)\,dt = 2xe^x.$$

b) Using Laplace transform solve the integral equation

$$\sin x = \int_0^x J_0(x-t)u(t)dt$$

c) Find eigen values and eigen functions of the homogeneous integral equation

$$u(x) = \lambda\int_0^1 \sin\pi x\cos\pi t\; u(t)dt.$$

d) Find first three iterated kernels of the kernel

$k(x, t) = (1 + x)(1 - t)$, $a = -1$, $b = 0$. **(4+4+3+3)**

3. a) Reduce the boundary value problem

$y''(x) + \lambda y(x) = x$, $y(0) = 0$, $y'(l) = 0$ into an integral equation

b) Solve the integral equation $\int_0^\infty F(t)\sin pt\,dt = \begin{cases} 1, & 0 \le p < 1; \\ 2, & 1 \le p < 2; \\ 0, & p \ge 2. \end{cases}$ **(8+6)**

4. a) Solve the homogeneous Fredholm integral equation of the second kind

$$u(x) = \lambda\int_0^\pi (\cos^2 x\cos 2t + \cos 3x\cos^3 t)\,u(t)\,dt$$

b) Prove that eigen functions of a symmetric kernel corresponding to different eigen values are orthogonal. **(8+6)**

5. a) Using Hilbert Schmidt theorem, solve the integral equation

$$u(x) = (x + 1)^2 + \int_{-1}^{1} (xt + x^2 t^2)u(t)\, dt$$

b) Show that m^{th} iterated kernel $k_m(x, t)$ satisfies the relation

$k_m(x,t) = \int_a^b k_p(x, y)\, k_{m-p}(y, t)\, dy$ where p is any positive integer less than m. **8+6)**

6. a) Find resolvent kernel and hence solve the integral equation

$$u(x) = f(x) + \lambda \int_0^1 e^{(x-t)} u(t)\, dt.$$

b) Solve the Abel's integral equation $x(x + 1) = \int_0^x \frac{u(t)}{\sqrt[3]{x-t}} dt$. **(7+7)**

7. a) By method of successive approximation solve the integral equation

$$u(x) = 1 + \int_0^x u(t)dt,\ u_0(x) = 0$$

b) Form an integral equation corresponding to differential equation

$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0$ with initial conditions $y(0) = 1,\ y'(0) = 0.$ **(7+7)**

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SLR-MA – 545

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
MATHEMATICS (Paper – XIX) (Old) (CGPA)
Elective – II : Operations Research

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q No. **2** are **compulsory**.*
*3) Attempt **any 3** questions from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Select correct alternative : **5**

1) The optimal solution for LPP

Maximize $Z = 2x_1 + 5x_2$,

Subject to constraint $2x_1 + 3x_2 \leq 15$, $x_2 \leq 5$, $x_1 \leq 2$, $x_1, x_2 \geq 0$ is given by

a) $x_1 = 2, x_2 = 0$ b) $x_1 = 0, x_2 = 5$
c) $x_1 = 2, x_2 = 5$ d) none of these

2) The quadratic form X^TQX is said to be negative semi-definite if

a) $X^TQX > 0$ b) $X^TQX < 0$
c) $X^TQX = 0$ d) $X^TQX \leq 0$

3) For a given LPP, if Z is a objective function then

a) Max Z = – Min Z b) Max Z = Min (– Z)
c) Max (– Z) = Max Z d) None of these

4) Which of the following method of solving QPP is based on modified simplex method ?

a) Wolfe's method b) Branch and bound method
c) Frant-Wolfe method d) Flectcher's method

P.T.O.

5) An optimal solution to an LPP

a) always corresponds to an extreme point of feasible region

b) always lies on the boundary of feasible region

c) always exists

d) none of these

b) Fill in the blanks : **5**

1) A game is said to be fair if _______________

2) Slack variables are used to convert the inequalities of the type _______________ into equations.

3) The competitors of the game are known as _______________

4) If an artificial variable is found in the optimum basis of an LPP, it implies _______________

5) If the given LPP is in its standard form, the primal dual pair is said to be _______________

c) State whether the following statements are **true** or **false**. **4**

1) The solution to a game by graphic method may or may not be same as obtained by analytic method.

2) One of the methods of handling degenerate LPPs is known as Charnes perturbation method.

3) Dual simplex methods is an alternative method to Big M method.

4) Dual simplex method always leads to degenerate basic feasible solution.

2. a) Answer the following : **6**

i) Prove that the dual of dual is primal.

ii) Define Linear Programming Problem and state its assumptions.

b) Write short notes one : **8**

i) Mixed strategy game.

ii) Artificial variables.

3. a) Let $S \subset \mathbb{R}^n$ be a closed convex set. Then prove that for any point y not in S, there is a hyper plane containing y such that S is contained in one of the open half spaces determined by the hyper plane. **6**

   b) Show that a basic feasible solution of the LPP is a vertex of the convex set of feasible solutions. **8**

4. a) Describe Gomory's method of solving an all Integer Linear Programming Problem (ILPP). **6**

   b) Use simplex method to solve : **8**

   Maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$

   subject to the constraints,

   $$4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20$$
   $$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$
   $$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$
   $$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

5. a) State and prove basic duality theorem. **6**

   b) Using artificial constraint, solve the following LPP by dual simplex method. **8**

   Maximize $Z = 2x_3$

   subject to the constraints,

   $-x_1 + 2x_2 - 2x_3 \geq 8$

   $-x_1 + x_2 + x_3 \leq 4$

   $2x_1 - x_2 + 4x_3 \leq 10$

   and $x_1, x_2, x_3 \geq 0$

6. a) Define a Quadratic Programing Problem and obtain Kuhn-Tucker conditions for the same. **6**

   b) Solve the Quadratic Programming Problem (QPP) by Beale's method. **8**

   Maximize $f(x) = \frac{1}{4}(2x_3 - x_1) - \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$

   subject to constraints

   $x_1 - x_2 + x_3 = 1$ and $x_1, x_2, x_3 \geq 0$

7. a) Explain the graphical method of solving $2 \times n$ and $m \times 2$ games. **8**

   b) Explain the maxmin and minimax principle used in game theory. **6**

SLR-MA – 546

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
MATHEMATICS (Paper – XX) (Old CGPA)
Probability Theory (Elective – III)

Day and Date : Saturday, 9-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Select correct alternative : **5**

1) If $\{X_n, n \geq 1\}$ are non-negative random variables then
 a) $E(\underline{\lim}\, X_n) = \underline{\lim}\, E(X_n)$ b) $E(\underline{\lim}\, X_n) \leq \underline{\lim}\, E(X_n)$
 c) $\underline{\lim}\, E(X_n) \leq E(\underline{\lim}\, X_n)$ d) none of these

2) If $\phi_X(t)$ is characteristic function of X then characteristic function of a + b X is
 a) $e^{ita}\,\phi_X(bt)$ b) $e^{-ita}\,\phi_X(bt)$
 c) $e^{itb}\,\phi_X(at)$ d) $e^{-itb}\,\phi_X(at)$

3) X is integrable if and only if
 a) X^+ integrable b) X^- integrable
 c) $|X|$ is integrable d) All the above

4) Let $\{A_n\}$ be an independent sequence of events such that $\sum_{n=1}^{\infty} A_n = \infty$ then
 a) $P(\overline{\lim}\, A_n) = 0$ b) $P(\overline{\lim}\, A_n) = 1$
 c) $P(\underline{\lim}\, A_n) = 0$ d) $P(\underline{\lim}\, A_n) = 1$

5) If X and Y are independent random variables then
 a) $E(X + Y) = E(X) + E(Y)$ b) $E(XY) = E(X)\,E(Y)$
 c) $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$ d) all the above

P.T.O.

b) Fill in the blanks : **5**

1) Minimal field containing A^C is __________
2) If Ω is finite sample space then the trivial field is __________
3) The distribution function of random variable X can be determined from characteristic function using __________ formula.
4) Let P(.) is a probability measure defined on (Ω, IF) then P(A) $\geq$ 0 is called ________ property of measure.
5) WLLN states that sample mean converges in __________ to population mean.

c) State whether the following statements are **true** or **false** : **4**

1) $|\phi_X(t)|$ is bounded by 1.
2) Convergence in probability always implies almost sure convergence.
3) Pairwise independence implies mutual independence.
4) If A is null set then μ(A) = 0.

2. a) Answer the following : **6**

i) If X is random variable. Prove that 1 – X is also a random variable.
ii) Define indicator function. With usual notations show that

$$I_{A \cap B}(\omega) = \text{Min}\{I_A(\omega), I_B(\omega)\}.$$

b) Write short notes on the following : **8**

i) Bernoulli's weak law of large numbers.
ii) Dominated convergence theorem.

3. a) Let $\{A_n\}$ and $\{B_n\}$ are two sequence of sets. Prove that

$$\overline{\lim}\,(A_n \cup B_n) = \overline{\lim}\,A_n \cup \overline{\lim}\,B_n$$

b) Find lim inf and lim sup of following sequence of sets.

i) $A_n = \left[3, 3 + \frac{5}{n}\right]$

ii) $A_n = \left(0, b + \frac{(-1)^n}{n}\right), b > 0$. **(6+8)**

4. a) Define a field. Examine for the class of finite or co-finite sets to be a field.

   b) Establish the continuity property of probability measure. **(7+7)**

5. a) Define mapping. Let X be a mapping defined on sample space Ω. Let A and $B \subset \Omega$ such that $A \cap B = \phi$. Prove or disprove : $X(A) \cap X(B) = \phi$.

   b) State and prove Borel-Cantelli lemma. **(7+7)**

6. a) Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

   b) Let $\{X_n, Y_n\}$, n = 1, 2, . . . be a sequence of random variables such that $|X_n - Y_n| \xrightarrow{P} 0$, $Y_n \xrightarrow{L} X$ then prove that $X_n \xrightarrow{L} X$. **(7+7)**

7. a) State inversion formula. Obtain the probability density function corresponding to the characteristic function $\phi_x(t) = e^{-\frac{t^2}{2}}$.

   b) State Lindeberg-Feller Central Limit Theorem (CLT) and deduce Liapunov's CLT as a special case. **(7+7)**
