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**M.Sc. (Semester – I) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**REAL ANALYSIS**

Day & Date: Monday, 18-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) The closed set includes all of its \_\_\_\_\_ points.
  - a) interior
  - b) limit
  - c) member
  - d) none of these
- 2) If A and B are open sets, then  $A \cup B$  is \_\_\_\_\_.
  - a) always open
  - b) always closed
  - c) may or not be open
  - d) neither open nor closed
- 3) A set is said to be closed, if \_\_\_\_\_.
  - a) it includes all of its interior points
  - b) if every point of set is its limit point
  - c) if it includes all of its limit points
  - d) none of these
- 4) A compact set is always \_\_\_\_\_.
  - a) bounded
  - b) closed
  - c) both (a) and (b)
  - d) none of these
- 5) A convergence limit for a sequence is \_\_\_\_\_.
  - a) necessarily unique
  - b) not necessarily unique
  - c) both (a) and (b)
  - d) none of these
- 6) If a set is open, then its compliment \_\_\_\_\_.
  - a) has to be open
  - b) may or may not be open
  - c) has to be closed
  - d) all of these
- 7) The set of natural numbers is \_\_\_\_\_.
  - a) bounded above
  - b) bounded below
  - c) both (a) and (b)
  - d) bounded
- 8) The finite union of finite sets is \_\_\_\_\_.
  - a) finite
  - b) countably infinite
  - c) uncountable
  - d) may be finite or countable
- 9) A point c is said to be extremum point of function f, if \_\_\_\_\_.
  - a)  $f'(c) = 0$
  - b)  $f(c) = 0$
  - c)  $f'(c) \neq 0$
  - d) none of these
- 10) The sequence  $S_n = \sin\left(\frac{2\pi}{n}\right), n \in N$  is \_\_\_\_\_.
  - a) convergent to 1
  - b) oscillatory
  - c) convergent to 0
  - d) none of these

- 11) The function  $f(x) = 2 - x + x^2$  has extrema at the point \_\_\_\_\_.  
a)  $\frac{1}{2}$   
b) 1  
c)  $\frac{1}{37}$   
d) None of these
- 12) A continuous function is \_\_\_\_\_.  
a) always differentiable  
b) always right continuous  
c) always bounded  
d) all of these
- 13) If A is finite set and  $A \cup B$  is countable set, then \_\_\_\_\_.  
a) B must be countable  
b) B may or may not be countable  
c) B is finite  
d) none of these
- 14) A geometric series with common ratio r converges, if \_\_\_\_\_.  
a)  $|r| > 1$   
b)  $|r| < 1$   
c)  $r = 1$   
d) all of these

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define and illustrate countable set.
- 2) Define and illustrate convergent sequence.
- 3) Define and illustrate compact set.
- 4) State and prove necessary condition for convergence of a series.
- 5) Define and illustrate concept of limit point.

**B) Write notes. (Any Two) 06**

- 1) Cauchy sequence
- 2) Mean value theorem
- 3) Geometric series/

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Check whether following series are convergent.  
i)  $\sum_{n=1}^{\infty} \frac{x^n}{n!}$   
ii)  $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$
- 2) Explain any two tests for convergence of a series.
- 3) Prove that the set  $[0,1]$  is uncountable.

**B) Answer the following questions. (Any One) 06**

- 1) Explain how to calculate Riemann integration of a continuous function.
- 2) Prove: Countable union of countable sets is countable.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Explain Lagrange’s method for obtaining constrained maxima or minima.
- 2) State and prove fundamental theorem on calculus.
- 3) Prove that a set is closed, if and only if its complement is open.

**B) Answer the following questions. (Any One) 04**

- 1) State Taylor’s theorem. Find the power series expansion for the following functions:  
a)  $f(x) = e^x$   
b)  $f(x) = e^{-x}$
- 2) Define radius of convergence. Also find it for the following power series.

$$1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots$$

**Q.5 Answer the following questions. (Any Two)**

- a) Find the stationary value of  $x^2 + y^2 + z^2$  subject to condition  $x^3 + y^3 + z^3 = 3a^3$ .
- b) Find upper Riemann integral and lower Riemann integral of  $f(x) = x^2$  over 1 to 2 and conclude whether the function is Riemann integrable.
- c) Explain limit superior and limit inferior of a sequence. Also give illustration.

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**M.Sc.(Semester - I) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**LINEAR ALGEBRA**

Day & Date: Tuesday, 05-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) Eigen values of an idempotent matrix are -
  - a) -1 or 1
  - b) 0 or 1
  - c) 2 or 1
  - d) None of these
- 2) Let A be a square matrix, then A is said to be nilpotent if-  
 for any positive integer k-
  - a)  $A^k = 0$
  - b)  $A^k = I$
  - c)  $A^k = -1$
  - d) None of these
- 3) For a matrix N with 5 rows and 3 columns,  $\rho(N)$  is rank of N then
  - a)  $\rho(N) \leq 5$
  - b)  $\rho(N) \geq 3$
  - c)  $\rho(N) \leq 3$
  - d)  $\rho(N) \geq 5$
- 4) Let B be any real matrix and A be its inverse then
  - a)  $BA = I$
  - b)  $AB = I$
  - c) both a) and b)
  - d) None of these
- 5) Eigen values of an upper triangular matrix are -
  - a) Its diagonal elements
  - b) off diagonal elements
  - c) all zero
  - d) None of the these
- 6) The vector  $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$  is an Eigen vector of the matrix  $\begin{bmatrix} 2 & 5 & 1 \\ 1 & 7 & -1 \\ 1 & 0 & 2 \end{bmatrix}$  then  
 corresponding Eigen value is -
  - a) 0
  - b) 1
  - c) 2
  - d) 3
- 7) Let V be a vector space of all functions  $f(x)$  where  $f: R \rightarrow R$   
 Then which of the following are subspace of V-
  - A. The constant function
  - B. The function with  $\lim_{x \rightarrow \infty} f(x) = 3$
  - C. Function with  $f(1) = 1$
  - D. Function with  $f(0) = 0$
  - a) A, B, C and D
  - b) A and D only
  - c) B,C and D only
  - d) B and D only
- 8) The column space of a non-singular matrix N of order 3 has dimension -
  - a) 3
  - b) less than 3
  - c) greater than 3
  - d) None of these



- 9) A vector space is closed under the operation of \_\_\_\_\_.  
a) addition and scalar multiplication    b) addition and subtraction  
c) Division and multiplication    d) None of these
- 10) Let  $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$  then  $A^{-1} =$  \_\_\_\_\_.  
a)  $\frac{1}{2} \begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$     b)  $\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix}$   
c)  $\frac{1}{2} \begin{bmatrix} 1 & 4 \\ -1 & -2 \end{bmatrix}$     d) None of these
- 11) Which of the following is an elementary row operation?  
a)  $R_i \leftrightarrow R_j$     b)  $k.R_i \rightarrow R_i, k \neq 0$   
c)  $R_i + k.R_j \rightarrow R_i, i \neq j$     d) All the above
- 12) M is negative definite matrix if and only if all of its Eigen values are -  
a) negative or positive    b) non positive  
c) negative    d) None of these
- 13) For a system of non-homogeneous equations  $Ax = b$ , it has solution if \_\_\_\_\_.  
a)  $\rho(A) = \rho(A : b)$     b)  $\rho(A) < \rho(A : b)$   
c)  $\rho(A) \neq \rho(A : b)$     d) None of these
- 14) The quadratic form  $2X_1^2 + X_2^2$  is -  
a) positive definite    b) negative definite  
c) positive semi definite    d) negative semi definite

**Q.2 a) Answer the following (any four):** **08**

- 1) Define algebraic and geometric multiplicity.
- 2) What is matrix of the quadratic form  $X_1^2 - 2X_2^2 - X_1X_1$ ?
- 3) Define Subspace. Give an illustration.
- 4) Define Kronekar product.
- 5) Define Eigen value and Eigen vector.

**b) Write Notes on (Any Two)** **06**

- 1) Elementary matrix operations
- 2) Row space and column space of a matrix
- 3) Singular value decomposition

**Q.3 a) Answer the following (Any two)** **08**

- 1) What is definiteness of a quadratic form?
- 2) Describe procedure of obtaining of system of Non-homogeneous linear equations?
- 3) How to obtain inverse of partitioned matrix?

**b) Answer the following (Any One):** **06**

- 1) Prove that any given quadratic form can be transformed to a quadratic form which contains only square terms.
- 2) Show that rank of product of any two real matrices does not exceeds rank of either of the matrix.

**Q.4 a) Answer the following (Any Two)** **10**

- 1) State and prove Cayley Hamilton theorem.
- 2) State and obtain necessary and sufficient condition for positive definiteness of a given quadratic form.
- 3) Define g-inverse of a matrix. Write procedure to obtain g-inverse.

**b) Answer the following (Any One):** **04**

- 1) Let  $X, Y$  and  $Z$  are linearly independent vectors. Examine whether  $U = X+Y, V = Y+Z$  and  $W = X+Z$  are linearly independent or not.
- 2) Write a short note on Spectral decomposition.

**Q.5 Answer the following (Any Two)** **14**

- a) Prove that any two linearly independent vectors in  $\mathbb{R}^2$  can form basis for  $\mathbb{R}^2$ .
- b) Obtain  $A^3$  and  $A^{-1}$  using Eigen value analysis, where  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
- c) Obtain orthonormal basis from the vectors  $a = (2, 0, 3), b = (1, 1, 0)$  and  $c = (0, 2, 1)$  using Gram-Schmidt process of orthonormalization.

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**M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**DISTRIBUTION THEORY**

Day & Date: Thursday, 07-11-2019  
Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) Let  $X$  be a  $N(\mu, \sigma^2)$  variable. Then distribution of  $e^x$  is \_\_\_\_\_.  
 a)  $N(0, \sigma^2)$                                       b) Lognormal  
 c) Standard normal                                  d) Half normal
- 2) Suppose  $X$  is  $B(n, p)$  random variable and define  $Y = 2X$ . Then distribution of  $Y = 2X$  is \_\_\_\_\_.  
 a)  $B(2n, p)$     b)  $B(n, 2p)$   
 c)  $B(2n, 2p)$                                       d) Not binomial
- 3) The p.g.f. of poisson ( $\lambda$ ) random variable is given by \_\_\_\_\_.  
 a)  $e^{-\lambda(1-s)}$                                       b)  $e^{-\lambda(s-1)}$   
 c)  $e^{\lambda(e^s-1)}$                                       d)  $e^{\lambda(e^s+1)}$
- 4) If  $z$  is standard normal variable then variance of  $z^2$  is \_\_\_\_\_.  
 a) 1    b) 2  
 c) 4    d) None of these
- 5) If  $X > 0$  then \_\_\_\_\_.  
 a)  $E[\log x] = \log[E(x)]$                               b)  $E[\log x] \geq \log[E(x)]$   
 c)  $E[\log x] \leq \log[E(x)]$                               d) None of these
- 6) m.g.f. of random variable  $X$  is  $\frac{(1+2e^t)^4}{81}$ , then mean of  $X$  is \_\_\_\_\_.  
 a)  $\frac{1}{2}$     b)  $\frac{8}{3}$   
 c) 2    d) None of these
- 7) Let  $X$  be a non-degenerate random variable and  $E(X) = 2$ . Then  $E(X^2)$  is \_\_\_\_\_.  
 a) Equal to 4    b) Less than 4  
 c) Greater than 4                                      d) None of these
- 8) If  $\mu'_1 = 2, \mu'_2 = 8$  and  $\mu_3 = 3$  then  $\mu'_3$  is \_\_\_\_\_.  
 a) 15    b) 25  
 c) 35    d) 45
- 9) If  $X$  has standard exponential distribution then \_\_\_\_\_.  
 a) mean = 2 variance                                  b) variance = 2 mean  
 c) mean = variance                                    d) None of these
- 10) Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution having p.d.f.  $f_X(x)$  and  $Y_1 \leq Y_2 \leq \dots \leq Y_n$  be the corresponding ordered sample. If p.d.f. of  $z$  is  $n[F_X(z)]^{n-1} f_X(z)$ , then  $z$  is \_\_\_\_\_.  
 a) Sample median                                      b) Sample range  
 c) Smallest observation                              d) Largest observation



**B) Answer the following questions. (Any One)**

04

- 1) Let  $X$  is a non-negative random variable with distribution function  $F$ . Show that

$$E(X) = \int_0^{\infty} [1 - F(x)] dx$$

- 2) Let  $X$  and  $Y$  are iid random variables with  $N(0, 1)$ . Find the distribution of  $Z = X + Y$ , using the result of convolution.

**Q.5 Answer the following questions. (Any Two)**

14

- a) Obtain the m.g.f. of multinomial distribution. Hence or otherwise find the variance-covariance matrix.
- b) Derive the joint p.d.f. of  $r^{\text{th}}$  and  $s^{\text{th}}$  order statistics based on a random sample from a continuous distribution with p.d.f.  $f(x)$  and e.d.f.  $F(x)$ .
- c) Let  $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ . Obtain the conditional distribution of  $Y$  given  $X$ .

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**M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**ESTIMATION THEORY**

Day & Date: Saturday, 09-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Multiple Choice Questions.****14**

- 1) Neyman factorization theorem is used to obtain
  - a) Sufficient statistic
  - b) Minimal sufficient statistic
  - c) Complete sufficient statistic
  - d) All of these
- 2) Minimal sufficient statistic is \_\_\_\_\_
  - a) Always sufficient
  - b) May not always exist
  - c) Is function of all sufficient statistics
  - d) All of these
- 3) Let  $X_1, X_2, X_3$  be iid from  $U(-\theta, \theta)$ .
  - a)  $\min(x_1, x_2, x_3)$  is sufficient statistic for  $\theta$
  - b)  $\max(x_1, x_2, x_3)$  is sufficient statistic for  $\theta$
  - c)  $(X_{(1)}, X_{(3)})$  is jointly sufficient statistic for  $\theta$
  - d)  $(x_1 + x_2 + x_3)$  is sufficient statistic for  $\theta$
- 4) Let  $X_1, X_2, X_3$  be iid from  $N(\theta, 1)$ . Then
  - a)  $\bar{X}$  and  $s^2$  is statistically independent
  - b)  $\bar{X}$  is UMVUE for  $\theta$
  - c)  $\bar{X}$  is complete sufficient statistic for  $\theta$
  - d) All of these
- 5) Let  $A(X)$  be the ancillary statistic then
  - a) Its distribution is free from parameter
  - b) Is unbiased estimator
  - c) Is complete sufficient statistic
  - d) None of these
- 6) Let  $f(x, \theta)$  be a probability distribution function belong to one parameter exponential family, then
  - a) UMVUE is function of complete sufficient statistic
  - b) MLE is function of sufficient statistic
  - c) Both (a) and (b)
  - d) Neither (a) nor (b)
- 7) Let  $X_1, X_2, \dots, X_n$  be iid from  $B(\theta)$ . Then conjugate prior distribution of  $\theta$  is
  - a) Gamma
  - b) Normal
  - c) Beta first kind
  - d) None of the above
- 8) Under squared error loss function Bays rule is
  - a) Posterior mean
  - b) Posterior standard deviation
  - c) Posterior median
  - d) None of these

- 9) Which of the following distribution belongs to exponential family of distributions?  
 a) *Laplace* ( $B, 1$ ) b) *Laplace* ( $\theta, \sigma$ )  
 c) *Laplace* ( $0, \theta$ ) d) None of these
- 10) Let  $X_1, X_2, \dots, X_n$  be iid from  $f(x, \theta)$ , then  $\prod_{i=1}^n f(x_i, \theta)$  is called \_\_\_\_\_  
 a) Maximum likelihood function b) Likelihood function  
 c) Marginal likelihood function d) Conditional likelihood function
- 11) Let  $X_1, X_2, \dots, X_n$  be iid from  $U(-\frac{\theta}{2}, \frac{\theta}{2})$ , then the maximum MLE for  $\theta$  is  
 a)  $\text{Max}(-x_{(1)}, x_{(n)})$  b)  $\text{Min}(-x_{(1)}, x_{(n)})$   
 c)  $x_{(n)}$  d)  $x_{(1)}$
- 12) Which of the following is not true  
 a) Unbiased estimator is always function of complete sufficient statistic  
 b) Unbiased estimator is not unique  
 c) Unbiased estimator is not always exist  
 d) Unbiased estimator may be absurd
- 13) Based on random sample of size  $n$  form truncated Poisson distribution with parameter  $\lambda$ , then  
 a) The maximum likelihood estimator is sample mean  
 b) Moment estimator is sample mean  
 c) Both (a) and (b)  
 d) Neither (a) nor (b)
- 14) Which of the following technique is used to obtain minimal sufficient statistic  
 a) Likelihood equivalence principle  
 b) Neyman factorization theorem  
 c) Basu's Lemma  
 d) None of these

- Q.2 A) Answer the following (Any Four) 08**  
 1) Describe concept Of Bayesian estimation.  
 2) Define UMVUE. Explain how to obtain it.  
 3) Explain the term minimal sufficient partition.  
 4) Let  $X_1, X_2, \dots, X_n$  be iid from  $P(\theta)$ . Obtain MLE for  $e^{-\theta}$   
 5) Explain in detail pitman family of distributions.
- B) Write Notes on (Any Two) 06**  
 1) Invariance property of maximum likelihood estimator.  
 2) Fisher information and fisher information matrix  
 3) Type of prior distributions
- Q.3 A) Answer the following (Any two) 08**  
 1) Show that negative binomial distribution belong to power series distribution  
 2) Describe method of scoring.  
 3) Describe Bhattacharya bound.
- B) Answer the following (Any One) 06**  
 1) Based on random sample of size  $n$  from exponential distribution with mean  $\theta$  develop  $C - R$  lower bound for UMVUE of  $\theta$ .  
 2) Prove or disprove MLE is not unique

**Q.4 A) Answer the following (Any Two) 10**

- 1) Describe the concept of completeness and bounded completeness.
- 2) Let  $X_1, X_2, \dots, X_n$  be random sample from distribution  $U(0, \theta)$  Show that  $\frac{n+1}{n} X_{(n)}$  is UMVUE for  $\theta$ .
- 3) Let  $X_1, X_2, \dots, X_n$  be random sample from distribution with pdf  $f(x, \theta) = \theta * (1 - \theta)^x, x = 0, 1, \dots, 0 < \theta < 1$  and prior distribution of  $\theta$  is  $U(0, 1)$ . Find posterior distribution of  $\theta$ .

**B) Answer the following (Any One) 04**

- 1) Let  $X_1, X_2, \dots, X_n$  be random sample from distribution with pdf  $f(x, \theta) = \theta e^{-\theta(x-\mu)}, x \geq \mu$ . Show that  $X_{(1)}$  and  $\sum X_i - X_1$  are independent.
- 2) State and prove Lehman-Scheffe theorem.

**Q.5 Answer the following (Any Two) 14**

- a) State and prove Fisher-Neyman factorization theorem in discrete case.
- b) State and Prove characterization property of UMUVE
- c) For the Bivariate population

$$f(x, y) = \binom{x}{y} (1-p)^{x-y} e^{-\lambda} \frac{\lambda^x}{x!}, 0 < p < 1, \lambda > 0, \quad y = 0, 1, \dots, x, x = 1, 2, \dots,$$

Find the moment estimator of  $(\lambda, p)$



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**M.Sc. (Semester – I) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**STATISTICAL COMPUTING**

Day & Date: Wednesday, 13-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing the correct alternatives given below. 14**

- 1) In MS-Excel, the \_\_\_\_\_ function returns the maximum value in a list of arguments.
  - a) large()
  - b) maximum()
  - c) max()
  - d) None of these
- 2) To obtain the arithmetic mean of arguments, the \_\_\_\_\_ command is used in MS-Excel.
  - a) average()
  - b) avg()
  - c) amean()
  - d) none of these
- 3) To obtain one sample from bivariate exponential distribution, we need to draw \_\_\_\_\_ exponential random numbers.
  - a) Three
  - b) Two
  - c) Twelve
  - d) One
- 4) The R-command to obtain value of pmf of Poisson(2) distribution at point 1 is \_\_\_\_\_.
  - a) ppois(2,1)
  - b) ppois( 1,2)
  - c) dpois(1,2)
  - d) none of these
- 5) In R, to obtain inverse of matrix A, the command used is \_\_\_\_\_.
  - a) inv(A)
  - b) invert(A)
  - c) in(A)
  - d) none of these
- 6) Congruential random number generator gives \_\_\_\_\_ random numbers.
  - a) True
  - b) Binomial
  - c) Pseudo
  - d) none of these
- 7) The distribution function of normal distribution follows \_\_\_\_\_ distribution.
  - a) chi-square
  - b) Normal
  - c) Beta
  - d) Uniform
- 8) In R, equality operator is given by \_\_\_\_\_.
  - a) =
  - b) ==
  - c) !=
  - d) neither (a) nor (b)
- 9) R-command for extracting second value of vector a is \_\_\_\_\_.
  - a) a2
  - b) a(2)
  - c) a[2]
  - d) none of these
- 10) In boot-strap technique \_\_\_\_\_ method is used for resampling.
  - a) Stratified
  - b) Systematic
  - c) SRSWR
  - d) SRSWOR

- 11) Addition of two independent binomial (10, 0.2) variates is \_\_\_\_\_.
  - a) binomial (20, 0.2)
  - b) Multinomial
  - c) binomial (10,0.2)
  - d) none of these
- 12) The \_\_\_\_\_ command is used to repeat same value in R.
  - a) rep()
  - b) repeat()
  - c) replicate()
  - d) all of these
- 13) The R-command to generate a random sample of size 5 from geometric (0.2) is \_\_\_\_\_.
  - a) rgeom(0.2,5)
  - b) rgeo(5,0.2)
  - c) rgeom(5,0.2)
  - d) none of these
- 14) Newton- Raphson method is used to \_\_\_\_\_.
  - a) Find roots of the equation  $f(x)=0$
  - b) Maximize a function  $f(x)$
  - c) Minimize a function  $f(x)$
  - d) Optimize a function  $f(x)$

**Q.2 A) Answer the following questions . (Any Four)**

**08**

- 1) Give an R-command to enter following matrix:

$$A = \begin{bmatrix} 3 & 1 & 9 \\ 2 & 5 & 7 \\ 1 & 6 & 8 \end{bmatrix}$$

- 2) State R-command to find inverse of following matrix:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & 2 \\ 1 & 2 & 8 \end{bmatrix}$$

- 3) State MS-Excel commands to calculate absolute value as well as rounded value of a number.
- 4) Write MINITAB command to obtain 10 random numbers from Bernoulli with success probability as 0.4.
- 5) Write -command to obtain:
  - i) Distribution function of binomial (3,0.8) at 2.3.
  - ii) Distribution function of Poisson (2) at 3.5.

**B) Write Notes. (Any Two)**

**06**

- 1) Jack-Knife estimator
- 2) Congruential random number generator
- 3) Commands to obtain covariance and correlation in R

**Q.3 A) Answer the following questions. (Any Two)**

**08**

- 1) How to carry out matrix operations in MS-Excel. Explain various commands for matrix operations.
- 2) Write down an algorithm and program for regula-falsi method.
- 3) Explain any two methods to check uniformity of random numbers.

**B) Answer the following questions. (Any One)**

**06**

- 1) Write an R-program to calculate factorial of a positive integer.
- 2) Explain various R-commands related with frequencies and cross tables.

- Q.4 A) Answer the following questions. (Any Two) 10**
- 1) Describe Monte-Carlo method to estimate  $\pi$ . Also write an algorithm for the same.
  - 2) State and prove the result to generate observations from geometric distribution.
  - 3) Write an algorithm to generate k observations from multinomial  $(n, p_1, p_2, p_3)$  distribution.
- B) Answer the following questions. (Any One) 04**
- 1) Write MINITAB macros to :
    - i) Generate 50 observations from Beta (3,4) distribution.
    - ii) Generate 40 observations from binomial with mean 5, variance 2.5.
  - 2) Explain the algorithms to generate random numbers from Binomial  $(n, p)$ .
- Q.5 Answer the following questions. (Any Two) 14**
- a) Explain the Newton-Raphson method.
  - b) Describe Simpson's rule to obtain a numerical integration.
  - c) Discuss bootstrap method of bias reduction. State clearly the assumptions, if any.

Seat  
No.Set **P**

**M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**PROBABILITY THEORY**

Day & Date: Monday, 04-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below.****14**

- 1) A  $\sigma$ -field is closed under \_\_\_\_\_.  
 a) complimentation  
 b) countable union  
 c) countable intersection  
 d) all of these
- 2) For a sequence  $\{A_n\}$  of sets, \_\_\_\_\_.  
 a)  $\overline{\lim} A_n = \underline{\lim} A_n$   
 b)  $\overline{\lim} A_n \subset \underline{\lim} A_n$   
 c)  $\underline{\lim} A_n \subset \overline{\lim} A_n$   
 d) None of these
- 3) The smallest field of subsets of  $\Omega$  contains \_\_\_\_\_ sets.  
 a) 3  
 b) 4  
 c) 2  
 d) cannot be predicted
- 4) The simple function is \_\_\_\_\_ linear combination of indicator of sets.  
 a) finite  
 b) arbitrary  
 c) any  
 d) none of these
- 5) If  $A$  and  $B$  are two subsets of  $\Omega$ , then  $P(A \cap B)$  \_\_\_\_\_.  
 a)  $= P(A) + P(B)$   
 b)  $< P(A) + P(B)$   
 c)  $> P(A) + P(B)$   
 d) None of these
- 6) The characteristic function of a random variable  $X$  degenerate at  $c$ , equals \_\_\_\_\_.  
 a)  $e^{itc}$   
 b)  $itc$   
 c)  $i(c + t)$   
 d)  $e^{-tc}$
- 7) A non-negative measure  $\mu$  defined on subsets of  $\Omega$  is said to be finite if and only if \_\_\_\_\_.  
 a)  $\mu(\Omega) = k < \infty$   
 b)  $\mu(\Omega) = 2$   
 c) there are finitely many subsets of  $\Omega$   
 d) all the these
- 8) Convergence in probability implies \_\_\_\_\_.  
 a) convergence in distribution  
 b) convergence in  $r^{\text{th}}$  mean  
 c) convergence in almost sure  
 d) None of these
- 9) If  $A$  contains finite number of elements, then set  $A^c$  is called as \_\_\_\_\_.  
 a) cofinite  
 b) slightly finite  
 c) nearly finite  
 d) None of these
- 10) Expectation of non-negative random variable follows \_\_\_\_\_.  
 a) linearity  
 b) scale preserving  
 c) both (a) and (b)  
 d) None of these

- 11) If  $P(\cdot)$  is a probability measure on  $(\Omega, F)$  and if  $P(A) = 1$ , then  $A$  is \_\_\_\_\_.
  - a)  $\Phi$
  - b)  $\Omega$
  - c) may or may not be  $\Omega$
  - d) None of these
- 12) A r.v.  $X$  is integrable, if and only if \_\_\_\_\_.
  - a)  $\sin X$  is integrable
  - b)  $X^2$  is integrable
  - c)  $|X|$  is integrable
  - d) None of these
- 13) If  $X \geq 0$  a.s., then  $E(X)$  \_\_\_\_\_.
  - a) can be negative
  - b)  $\geq 0$
  - c) = 0
  - d) none of these
- 14) Which of the following is an elementary random variable?
  - a) Bernoulli r.v.
  - b) Geometric r.v.
  - c) binomial r.v.
  - d) None of these

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define probability measure.
- 2) Define Lebesgue measure.
- 3) Define  $\sigma$ -field.
- 4) State Liaponove’s Theorem on CLT
- 5) Define indicator function.

**B) Write notes. (Any Two) 06**

- 1) Prove that collection of sets whose inverse images belong to a  $\sigma$ -field, is a also a  $\sigma$ -field.
- 2) Prove or disprove: Intersection of two fields is a field.
- 3) Discuss the construction of  $\sigma$ -field induced by r.v.  $X$ .

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Define conditional probability measure. Show that it is also a probability measure.
- 2) Prove or disprove: Mapping preserves all set relations.
- 3) Prove or disprove: Arbitrary union of fields is a field.

**B) Answer the following questions. (Any One) 06**

- 1) Discuss limit superior and limit inferior of a sequence of sets. Find the same for sequence  $\{A_n\}$ , where  $A_n = \left(0, 3 + \frac{(-1)^n}{n}\right), n \in N$
- 2) State and prove Fatou’s lemma.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) State the constructive definition of arbitrary random variable using simple random variable. Justify.
- 2) Define convergence in probability and convergence in distribution. Prove or disprove: convergence in distribution implies convergence in probability.
- 3) Define expectation of simple random variable. If  $X$  and  $Y$  are simple random variables, prove the following:
  - i)  $E(X + Y) = E(X) + E(Y)$
  - ii)  $E(cX) = c E(X)$ , where  $c$  is a real number
  - iii) If  $X > 0$  a.s., then  $E(X) > 0$ .

**B) Answer the following questions. (Any One) 04**

- 1) Define the characteristic function of a r.v. and find the same for exponential distribution.
- 2) Prove any three properties of indicator function.

**Q.5 Answer the following questions. (Any Two)**

- a) State and prove monotone convergence theorem.
- b) Prove that if  $\{B_n\}$  converges to  $B$ , then  $P(B_n)$  also converges to  $P(B)$ .
- c) Show that there are classes which are field but not  $\sigma$ -field.

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**M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**STOCHASTIC PROCESSES**

Day & Date: Wednesday, 06-11-2019  
 Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) Addition of two independent Poisson processes is \_\_\_\_\_.
  - a) Binomial process
  - b) compound Poisson process
  - c) Poisson process
  - d) all of these
- 2) If state j is aperiodic persistent non-null then as  $n \rightarrow \infty$ ,  $P_{jj}^{(n)} \rightarrow$  \_\_\_\_\_.
  - a) 1
  - b) 0
  - c)  $1/\mu_{jj}$
  - d) Limit does not exist
- 3) All the entries of transition probability matrix (TPM) are always \_\_\_\_\_.
  - a) Positive
  - b) Non-negative
  - c) Integer
  - d) None of these
- 4) The process  $\{X_n\}$ , where  $X_n$  = number of patients in a hospital on  $n^{\text{th}}$  day, is an example of \_\_\_\_\_ stochastic process.
  - a) discrete time continuous state space
  - b) discrete time discrete state space
  - c) continuous time continuous state space
  - d) continuous time discrete state space
- 5) Recurrent state is also called as \_\_\_\_\_.
  - a) ergodic
  - b) persistent
  - c) transient
  - d) None of these
- 6) In a Branching process if  $E X_1 = m$ , then  $E X_n =$  \_\_\_\_\_.
  - a) n
  - b)  $m^n$
  - c)  $n^m$
  - d) None of these
- 7) For a symmetric random walk, probability 'p' of positive jump is \_\_\_\_\_.
  - a) 0.25
  - b) 0.5
  - c) 1
  - d) None of these
- 8) If  $\{N(t)\}$  is a Poisson process, then the inter-arrival times follow \_\_\_\_\_.
  - a) beta distribution of second kind
  - b) Poisson distribution
  - c) binomial distribution
  - d) exponential distribution
- 9) For a aperiodic state, the period is \_\_\_\_\_.
  - a) 0
  - b) not defined
  - c) 1
  - d) None of these





**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) State and prove Chapman-Kolmogorov equations.
- 2) Define stationary distribution. Obtain the same for the Markov chain  $\{X_n, n \geq 0\}$  with state space  $S = \{1,2,3\}$  and TPM as-

$$P = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.2 & 0.4 & 0.4 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

- 3) Discuss Yule-Furry process. Obtain the expression for  $P_n(t)$ .

**B) Answer the following questions. (Any One) 04**

- 1) Describe Pure birth process as well as birth and death process.
- 2) Define:
  - i) Ergodic state
  - ii) Transient State
  - iii) Absorbing state
  - iv) Period of a state

**Q.5 Answer the following questions. (Any Two) 14**

- a) Prove that persistency is a class property.
- b) Obtain Kolmogorov differential equations for birth and death process.
- c) Discuss Gambler's ruin problem in detail.

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**M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019**  
**Statistics**

**THEORY OF TESTING OF HYPOTHESES**

Day & Date: Friday, 08-11-2019  
Time: 11:30 AM To 02:00 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below.**

14

- 1) If  $\alpha$  and  $\beta$  are probability of Type I and Type II errors. Which one of the following is the probability of rejecting  $H_0$  when  $H_1$  is true?
  - a)  $\alpha$
  - b)  $1-\alpha$
  - c)  $\beta$
  - d)  $1-\beta$
- 2) The p.d.f.  $f(x) = \frac{1}{2}e^{-|x-\theta|}$ ,  $-\infty < x < \infty$ , has MLR in \_\_\_\_\_.
  - a)  $x^2$
  - b)  $|x|$
  - c)  $x$
  - d)  $-x$
- 3) For comparing two test functions, which of the following measure is appropriate?
  - a) Size of test
  - b) Power of test
  - c) Variance of underlying test statistic
  - d) Unbiasedness of the test statistic
- 4) For goodness of fit test, the value of  $\chi^2$  statistic is zero if and only if \_\_\_\_\_.
  - a)  $\sum O_i = \sum E_i$
  - b)  $\sum O_i^2 = \sum E_i^2$
  - c)  $O_i = E_i$  for all  $i$
  - d) None of these
- 5) Test with Neyman structure is a \_\_\_\_\_.
  - a) Similar test
  - b) Subset of similar tests
  - c) Not a subset of similar tests
  - d) None of these
- 6) On the basis of single observantion  $X$  from  $U(0, \theta)$  distribution, the critical region for testing  $H_0: \theta = 1$  against  $H_1: \theta = 2$  is defined as  $\{0.5 < X < 2\}$ . Then power if the test is \_\_\_\_\_.
  - a) 0.25
  - b) 0.50
  - c) 0.75
  - d) 0.90
- 7) Let  $X_1, X_2, \dots, X_n$  be iid  $N(\theta, 1)$ . Let  $H_0: \theta = \theta_0$  and  $H_1: \theta \neq \theta_0$ . The UMPU level  $\alpha$  test rejects  $H_0$  iff \_\_\_\_\_.
  - a)  $\bar{X} > C_1$
  - b)  $\bar{X} < C_2$
  - c)  $C_1 < \bar{X} < C_2$
  - d)  $\bar{X} < C_1$  or  $\bar{X} > C_2$
- 8) A test function  $\phi(x) \equiv 0.5$  for all  $x$ , has power \_\_\_\_\_.
  - a) 1
  - b) 0
  - c) 0.5
  - d) None of these

- 9) Let  $H_1: \mu = 5$ , where  $\mu$  is mean of normal population from which sample is taken.  
 $H_2$  : population follows standard normal distribution.  
 a)  $H_1$  is simple and  $H_2$  is simple  
 b)  $H_1$  is simple and  $H_2$  is composite  
 c)  $H_1$  is composite and  $H_2$  is simple  
 d)  $H_1$  is composite and  $H_2$  is composite
- 10) A family of  $U(0, \theta)$  distribution has MLR in \_\_\_\_\_ when sample of size  $n$  is available from  $U(0, \theta)$ .  
 a)  $\bar{X}$     b)  $X_{(1)}$   
 c)  $X_{(n)}$     d) None of these
- 11) A test for testing  $H_0$  against  $H_1$  is called level  $\alpha$  test if \_\_\_\_\_.  
 a) Size of test does not exceeds  $\alpha$   
 b) Size of test is exactly equal to  $\alpha$   
 c) Hypothesis of the test is simple hypothesis  
 d) The test is unbiased
- 12) For  $N(\theta, 1)$  distribution, pivotal quantity for confidence interval of  $\theta$  based on  $X_1, X_2, \dots, X_n$  is \_\_\_\_\_.  
 a)  $n \bar{X}$     b)  $\sqrt{n} \bar{X}$   
 c)  $n(\bar{X} - \theta)$                                       d)  $\sqrt{n}(\bar{X} - \theta)$
- 13) A UMP test is \_\_\_\_\_.  
 a) Always exists                                      b) Biased test  
 c) Unbiased test                                      d) None of these
- 14) The acceptance region of UMP size  $\alpha$  test leads to \_\_\_\_\_ confidence set.  
 a) UMA    b) UMAU  
 c) Biased    d) Unbiased

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define simple hypothesis and composite hypothesis. Give one example for each.
- 2) Define pivotal quantity. Give an example.
- 3) Define U statistic and give an example.
- 4) Define UMA confidence interval.
- 5) Define likelihood ratio test.

**B) Answer the following questions. (Any Two) 06**

- 1) Test for independence of attributes.
- 2) Mann-Whitney test
- 3) Type I and type II errors

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Define monotone likelihood ratio (MLR) of probability distributions. Show that exponential distribution with mean  $\theta$  possess MLR property.
- 2) Prove or disprove: MP test is not unique
- 3) Use N-P lemma to test  $H_0: \theta = 0$  against  $H_1: \theta = 1$  on the basis of random sample of size  $n$  from  $N(\theta, 1)$  distribution.

**B) Answer the following questions. (Any One) 06**

- 1) Let  $X_1, X_2, \dots, X_n$  be a random sample from  $U(0, \theta)$  distribution. Obtain  $(1 - \alpha)$  level shortest length confidence interval for  $\theta$ .
- 2) Explain the concept of unbiased test. Examine whether MP test is necessarily unbiased.

- Q.4 A) Answer the following questions. (Any Two) 10**
- 1) State and prove a necessary condition under which a UMP size  $\alpha$  similar test is UMPU test.
  - 2) Derive the relationship between UMA confidence set and UMP test.
  - 3) Let  $X_1, X_2, \dots, X_n$  are iid  $N(\theta, \sigma^2)$ , where  $\sigma^2$  is known. Show that UMP test does not exist for testing  $H_0: \theta = \theta_0$  and  $H_1: \theta \neq \theta_0$
- B) Answer the following questions. (Any One) 04**
- 1) Derive LRT for testing  $H_0: \theta = \theta_0$  and  $H_1: \theta \neq \theta_0$  based on a sample of size  $n$  from  $N(\theta, 1)$  distribution.
  - 2) State the generalized Neyman-Pearson lemma. Also explain in detail any one of its application.
- Q.5 Attempt any two of the following questions. (Any Two) 14**
- 1) Obtain the UMPU level  $\alpha$  test for testing  $H_0: \theta = \theta_0$  against  $H_1: \theta \neq \theta_0$  based on  $N(\theta, \sigma^2)$ , where  $\sigma^2$  is known for a sample of size  $n$ .
  - 2) Describe  $\chi^2$  test for goodness of fit.
  - 3) Let  $X \sim B(6, \theta)$ .  $H_0: \theta = \frac{1}{2}$ ,  $H_1: \theta = \frac{3}{4}$ . Compute the probabilities of type I and type II errors when test is given by reject  $H_0$  if  $X = 0, 6$ .

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**M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**SAMPLING THEORY**

Day & Date: Monday, 11-11-2019  
Time: 11:30 AM To 02:00 PM

Max. Marks: 70

- Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) If a heterogeneous population can be easily divided into sub populations with relatively small variability between the subpopulations then appropriate sampling design is \_\_\_\_\_.  
 a) Stratified  
 b) Two stage  
 c) Systematic  
 d) Cluster
  
- 2) In SRSWOR, the probability that a particular unit will be selected at  $r^{\text{th}}$  draw is \_\_\_\_\_.  
 a)  $\frac{r}{N}$   
 b)  $\frac{1}{N - r}$   
 c)  $\frac{1}{N}$   
 d)  $\frac{1}{N - r + 1}$
  
- 3) In a linear systematic sampling with interval 40 from a population of 1000 units, the probability that a specified units is included in the sample is \_\_\_\_\_.  
 a)  $\frac{1}{35}$   
 b)  $\frac{1}{40}$   
 c)  $\frac{25}{40}$   
 d)  $\frac{1}{1000}$
  
- 4) A population of size  $N = 5$  units has mean  $\bar{Y}_N = 12$  and  $S^2 = 100$ . A simple random sample of size  $n = 2$  units is drawn without replacement and sample mean is denoted by  $\bar{Y}_n$ . Then  $E[\bar{Y}_n^2]$  is \_\_\_\_\_.  
 a) 30  
 b) 50  
 c) 144  
 d) 174
  
- 5) In simple random sampling, the ratio estimator is \_\_\_\_\_.  
 a) Always biased  
 b) Always unbiased  
 c) Minimum variance unbiased  
 d) None of these
  
- 6) In SRSWR scheme, the variance of sample mean is given by \_\_\_\_\_.  
 a)  $\left(\frac{N-1}{N}\right)\frac{\sigma^2}{n}$   
 b)  $\frac{\sigma^2}{n}$   
 c)  $\left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n}$   
 d)  $\left(\frac{N-1}{N}\right)\sigma^2$
  
- 7) Stratified sampling is more precise than the systematic sampling if serial correlation coefficients are \_\_\_\_\_.  
 a) Positive  
 b) Negative  
 c) Nearly equal to one  
 d) Equal to zero

- 8) Non sampling errors occurs in \_\_\_\_\_.
- a) Only sample surveys                      b) Only complete enumeration  
c) Both a and b                                d) None of these
- 9) A city is divided into 100 non-overlapping blocks. Ten blocks are selected at random and completely enumerated. The procedure adopted is \_\_\_\_\_.
- a) Systematic sampling                      b) Double sampling  
c) Cluster sampling                            d) Stratified sampling
- 10) In sampling with probability proportional to size, the units are selected with probability proportional to \_\_\_\_\_.
- a) Size of the unit                              b) Size of the sample  
c) Population size                              d) None of these
- 11) The census Bureau in India takes a complete population count at every \_\_\_\_\_ years.
- a) 5    b) 10  
c) 12    d) None of these
- 12) Simple regression estimator of population mean is given by \_\_\_\_\_.
- a)  $\bar{X} + b(\bar{x} - \bar{y})$                                   b)  $\bar{y} + b(\bar{X} - \bar{x})$   
c)  $\bar{x} + b(\bar{X} - \bar{y})$                                   d)  $\bar{X} + b(\bar{y} - \bar{x})$
- 13) If n units are selected in a sample from N population units, the sampling fraction is \_\_\_\_\_.
- a)  $\frac{1}{n}$     b)  $\frac{1}{N}$   
c)  $\frac{n}{N}$     d)  $\frac{n-1}{N}$
- 14) Under Neyman allocation, the sample size for  $i^{th}$  stratum is proportional to \_\_\_\_\_.
- a)  $N_i S_i$     b)  $N_i S_i^2$   
c)  $N_i$     d)  $\frac{N_i}{S_i}$

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Give advantages of sampling method over census method.
- 2) Specify proportional allocation in stratified sampling.
- 3) Define probability proportional to size (PPS) sampling.
- 4) Distinguish between ratio and regression estimators.
- 5) Describe Murthy's unordered estimator.

**B) Write short notes. (Any Two) 06**

- 1) Midzuno system of sampling
- 2) Non-sampling errors
- 3) Circular systematic sampling

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Describe a procedure for obtaining a sample of size n from a population of size N using SRSWOR method.
- 2) Describe cumulative total method for PPS sampling.
- 3) Define a two-stage sampling design and give a practical situation where such a design can be used.

**B) Answer the following questions. (Any One) 06**

- 1) Derive the sampling variance of the systematic sample mean in terms of intraclass correlation.
- 2) Define Horvitz-Thompson estimator for the population total. Show that it is unbiased and obtain an unbiased estimator of its variance.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) In SRSWOR of  $n$  clusters each containing  $M$  elements from a population of  $N$  clusters. Obtain mean and variance of estimator of sample mean.
- 2) Explain the benefits of stratifying a population before sampling. Derive the optimum allocation for the sample size assuming a linear cost function.
- 3) In SRSWOR, show that the sample mean  $\bar{y}$  is unbiased for population mean. Obtain the sampling variance of  $\bar{y}$ .

**B) Answer the following questions. (Any One) 04**

- 1) In SRSWOR, show that the probability of drawing a specified unit at every draw is same.
- 2) Define PPSWR sampling design. Explain Lahiri's method for drawing a PPSWR sample.

**Q.5 Answer the following questions. (Any Two) 14**

- 1) Define ratio estimator and derive the approximate expression for bias. Assume SRSWOR scheme.
- 2) Discuss Hansen-Hurwitz technique in the presence of non-response in surveys.
- 3) Define systematic sampling. Discuss situations when systematic sampling is more efficient than SRSWOR.

Seat No.	
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**M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**ASYMPTOTIC INFERENCE**

Day & Date: Monday, 18-11-2019  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) The criterion used to choose between two consistent estimators is \_\_\_\_\_.
  - a) Smallness of mean
  - b) Smallness of variance
  - c) Smallness of mean squared error
  - d) None of these
- 2)  $\{U(o, \theta), \theta > 0\}$  = is \_\_\_\_\_.
  - a) one parameter exponential family
  - b) cramer family
  - c) both (a) and (b)
  - d) neither (a) nor (b)
- 3) If  $T_n$  is consistent for  $\theta$  then  $g(T_n)$  is consistent for  $g(\theta)$  if \_\_\_\_\_.
  - a)  $g$  is linear function
  - b)  $g$  is continuous function
  - c)  $g$  is differentiable function
  - d) none of these
- 4) Given a random sample of size  $n$  from  $N(\theta, 1)$ , the estimator  $\bar{X}_n$  is \_\_\_\_\_ for  $\theta$ .
  - a) unbiased
  - b) consistent
  - c) CAN
  - d) all the above
- 5) The test used to investigate the homogeneity of variances of several normally distributed populations is \_\_\_\_\_.
  - a) Rao test
  - b) Bartlett test
  - c) Pearson test
  - d) Wald test
- 6) Kullback - Leibie information index \_\_\_\_\_.
  - a)  $I(\theta, \theta_0) < 0$
  - b)  $I(\theta, \theta_0) > 0$
  - c)  $I(\theta, \theta_0) \geq 0$
  - d)  $I(\theta, \theta_0) = 0$
- 7) In case of  $U(0, \theta), \theta > 0$  the MLE of  $\theta$  is \_\_\_\_\_.
  - a) unbiased and consistent
  - b) asymptotically unbiased and consistent
  - c) unbiased but not consistent
  - d) asymptotically unbiased but not consistent
- 8) For distribution belonging to one parameter exponential family, moment estimator of  $\theta$  based on sufficient statistic is CAN for  $\theta$  with asymptotic variance \_\_\_\_\_.
  - a)  $n I(\theta)$
  - b)  $\frac{1}{nI(\theta)}$
  - c)  $I(\theta)$
  - d)  $\frac{1}{I(\theta)}$



- 9) Variance stabilizing transformation for poisson population is \_\_\_\_\_.  
 a) square root  
 b) logarithmic  
 c)  $\sin^{-1}$   
 d)  $\tan^{-1}$
- 10) If  $T_n$  is consistent estimator of  $\theta$  then  $e^{T_n}$  is \_\_\_\_\_.  
 a) unbiased estimator of  $e^\theta$   
 b) consistent estimator of  $e^\theta$   
 c) MVU estimator of  $e^\theta$   
 d) none of the above
- 11) Let  $x_1, x_2, \dots, x_n$  be iid with  $E(x_i^2) = V(x_i) = \sigma^2$  then asymptotic distribution of  $\bar{X}_n$  is \_\_\_\_\_.  
 a)  $N(0,1)$   
 b)  $N(0, \sigma^2)$   
 c)  $N\left(0, \frac{1}{n}\right)$   
 d)  $N\left(0, \frac{\sigma^2}{n}\right)$
- 12) The sample median is consistent estimator for  $\theta$  in the case of \_\_\_\_\_.  
 a)  $N(\theta, 1)$   
 b)  $U(\theta - 1, \theta + 1)$   
 c)  $Laplace(\theta, 1)$   
 d) all the above
- 13) Let  $x_1, x_2, \dots, x_n$  be iid  $N(\mu, 1)$ . Then asymptotic distribution of sample median  $M_n$  is \_\_\_\_\_.  
 a)  $N\left(\mu, \frac{\pi}{n}\right)$   
 b)  $N\left(\mu, \frac{\pi}{2n}\right)$   
 c)  $N\left(\mu, \frac{\pi^2}{4n}\right)$   
 d)  $N\left(\mu, \frac{1}{n}\right)$
- 14) With sufficiently large sample size with probability close to one, the likelihood equation admits \_\_\_\_\_.  
 a) unique consistent solution  
 b) two consistent solution  
 c) more than two consistent solutions  
 d) none of these
- Q.2 A) Answer the following questions. (Any Four) 08**  
 1) Define strong consistency.  
 2) Define Rao's score test.  
 3) Define BAN estimator.  
 4) Define multiparameter exponential family.  
 5) Define asymptotic relative efficiency.
- B) Write Notes. (Any Two) 06**  
 1) Super efficient estimator  
 2) CAN estimation in multiparameter exponential family  
 3) Bartlett's test for homogeneity of variances
- Q.3 A) Answer the following questions. (Any Two) 08**  
 1) Show that sample variance is consistent estimator of population variance, if it exists.  
 2) Show that sample distribution function at a given point is CAN for the population distribution function at the same point.  
 3) Let  $x_1, x_2, \dots, x_n$  be iid from exponential distribution with location parameter  $\theta$ . Examine whether  $x_{(1)}$  is consistent estimator for  $\theta$ .

- B) Answer the following questions. (Any One) 06**
- 1) Describe variance stabilizing transformation for poisson population.
  - 2) Let  $x_1, x_2, \dots, x_n$  be iid  $B(1, \theta)$ . Show that  $\bar{X}_n$  is CAN for  $\theta$ . Let  $\psi(\theta) = \theta(1 - \theta)$ . Show that  $\bar{X}_n(1 - \bar{X}_n)$  is CAN for  $\psi(\theta)$  for all values of  $\theta$  except  $\theta = \frac{1}{2}$ . What is asymptotic distribution of  $\bar{X}_n(1 - \bar{X}_n)$  at  $\theta = \frac{1}{2}$ ?
- Q.4 A) Answer the following questions. (Any Two) 10**
- 1) In case of one parameter exponential family, show that moment estimator based on sufficient statistic is CAN for the parameter.
  - 2) Let  $x_1, x_2, \dots, x_n$  be iid with distribution having p.d.f.  $f(x, \theta) = \frac{\theta}{x^{\theta+1}}$ ,  $x > 1, \theta > 0$ . Obtain CAN estimator of  $\theta$ .
  - 3) Let  $x_1, x_2, \dots, x_n$  be iid from  $N(\theta, \theta)$ , for  $\theta > 0$ . Obtain  $100(1-\alpha)\%$  confidence interval for  $\theta$  using variance stabilizing transformation.
- B) Answer the following questions. (Any One) 04**
- 1) Explain with illustration that the MLE need not be CAN.
  - 2) Let  $x_1, x_2, \dots, x_n$  be iid exponential with mean  $\theta$ . Obtain consistent estimator for first and third quartile of the distribution.
- Q.5 Answer the following questions. (Any Two) 14**
- a) Under Cramer - Huzurbazar regularity conditions, show that the likelihood equation admits a solution which is consistent.
  - b) Let  $x_1, x_2, \dots, x_n$  be a random sample of size  $n$  from  $N(\mu, \sigma^2)$ . Obtain MLE of  $(\mu, \sigma^2)$ . Show that it is CAN for  $(\mu, \sigma^2)$ . Obtain its asymptotic variance covariance matrix.
  - c) Derive the asymptotic distribution of likelihood ratio statistic.

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**M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**MULTIVARIATE ANALYSIS**

Day & Date: Tuesday, 05-11-2019  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) Generalised variance is \_\_\_\_\_ of covariance matrix.
  - a) trace
  - b) determinant
  - c) trace+ determinant
  - d) none of these
- 2) The mean vector of a random vector  $(X_1 X_2)$  is  $(3, 5)$ , then the mean vector of  $(X_1 + 2X_2, 2X_1 - X_2)$  is \_\_\_\_\_.
  - a)  $(3, 5)$
  - b)  $(13, 5)$
  - c)  $(13, 11)$
  - d)  $(13, 1)$
- 3) Principal components are \_\_\_\_\_.
  - a) orthogonal
  - b) uncorrelated
  - c) independent
  - d) all of these
- 4) For a multivariate normal random vector, the variance-covariance matrix is always \_\_\_\_\_.
  - a) square matrix
  - b) non-negative definite
  - c) symmetric
  - d) all of these
- 5) If  $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$ , then for a vector  $\underline{a}$ , the variable  $\underline{a}'\underline{X}$  follows which distribution?
  - a)  $N_p(\underline{\mu}, \underline{\Sigma})$
  - b)  $N_p(\underline{\mu}, n\underline{\Sigma})$
  - c)  $N_p(\underline{\mu} - \frac{1}{n}\underline{\Sigma})$
  - d) none of these
- 6) The \_\_\_\_\_ distribution is a multivariate generalization of chi-square distribution.
  - a) Multivariate Normal
  - b) Hotelling's  $T^2$
  - c) Wishart distribution
  - d) None of these
- 7) Statistical techniques that focus upon bringing out the structure of simultaneous relation among three or more variables are called \_\_\_\_\_ analysis.
  - a) bivariate
  - b) parametric
  - c) multivariate
  - d) non-parametric
- 8) A canonical correlation cannot be negative, because \_\_\_\_\_.
  - a) we take only positive eigen values
  - b) it is generalisation of multiple correlation
  - c) we take only positive square root
  - d) we rejected negative value
- 9) In factor analysis, if there are k variables and m factors, then \_\_\_\_\_.
  - a)  $k < m$
  - b)  $m < k$
  - c)  $m = k$
  - d) none of these

- 10) Based on a random sample of size  $n$  from  $N_p(\mu, \Sigma)$ , the distribution of  $\bar{X}$  is \_\_\_\_\_.
- a)  $N_p(\mu, \Sigma)$
  - b)  $N_p(\mu, \frac{1}{n}\Sigma)$
  - c)  $N_p(\mu, \frac{1}{n-1}\Sigma)$
  - d) none of these
- 11) While applying \_\_\_\_\_ clustering algorithm, the distance between two clusters is taken to be the smallest distance between observations from two clusters.
- a) single linkage
  - b) average linkage
  - c) complete linkage
  - d) none of these
- 12) Among the principal components, the \_\_\_\_\_ principle component has largest variance.
- a) first
  - b) last
  - c)  $(p/2)^{\text{th}}$
  - d) none of these
- 13) Let  $\underline{X}$  is multivariate normal, then  $\underline{a}'\underline{X}$  is univariate normal, only if \_\_\_\_\_.
- a)  $\underline{a}$  is unit vector
  - b)  $\underline{a}$  is zero vector
  - c) for all  $\underline{a}$
  - d) none of these
- 14) Let  $X \sim N_p(\mu, \Sigma)$  then variance of  $AX$  is \_\_\_\_\_.
- a)  $A \Sigma A'$
  - b)  $A' \Sigma A$
  - c)  $AA \Sigma$
  - d) none of these

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define multivariate normal distribution.
- 2) State Wishart density function.
- 3) Define multiple correlation coefficients.
- 4) Define Hotelling- $T^2$  statistics.
- 5) Define variance covariance matrix.

**B) Write notes. (Any Two) 06**

- 1) Mahalanobis distance
- 2) Generalised variance
- 3) Characteristic function of Wishart distribution

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Let vector  $X = (X_1, X_2, \dots, X_p)'$  be distributed according to  $N_p(\mu, \Sigma)$ . Then find marginal distribution of  $X_1$ .
- 2) Show that two p-variate normal vectors  $X_1$  and  $X_2$  are independent iff  $Cov(X_1, X_2) = 0$
- 3) If vector  $X$  is distributed according to  $N_p(\underline{\mu}, \Sigma)$  and  $A$  is a  $k \times p$  matrix, then find distribution of  $AX$ .

**B) Answer the following questions. (Any One) 06**

- 1) Write short notes on singular and non-singular normal distribution.
- 2) Explain the technique of principle component.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) State and prove additive property of Wishart distribution.
- 2) How one can find correlation between two multivariate normal random vectors? Explain in brief.
- 3) Explain in brief the idea of factor analysis.

**B) Answer the following questions. (Any One)**

04

- 1) Derive the moment generating function of  $N_p(\underline{\mu}, \underline{\Sigma})$  distribution.
- 2) Let  $A \sim W_p(\underline{\mu}, \underline{\Sigma})$  and  $\underline{a}$  be a  $(p \times 1)$  vector which is independently distributed.

Then obtain the distribution of  $\frac{\underline{a}' A \underline{a}}{\underline{a}' \underline{\Sigma} \underline{a}}$

**Q.5 Answer the following questions. (Any Two)**

14

- 1) Explain method of clustering. What is meant by agglomerative clustering and divisive clustering? Also explain single linkage and complete linkage.
- 2) Discuss the problem of discrimination for multivariate observation. Also explain costs associated with it.
- 3) In usual notations, for  $N_p(\underline{\mu}, \underline{\Sigma})$ , show that  $X$  and  $S$  are maximum likely estimators of  $\underline{\mu}$  and  $\underline{\Sigma}$  respectively.

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**M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019  
Statistics**

**PLANNING AND ANALYSIS OF INDUSTRIAL EXPERIMENTS**

Day & Date: Thursday, 07-11-2019  
Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 Fill in the banks by choosing correct alternatives given below. 14**

- 1) If there are six factors each at two levels and are conducted in two replications, then error degrees of freedom are \_\_\_\_\_.  
a) 0  
b) 39  
c) 64  
d) 128
- 2) In the field experimentation, when experimental material is heterogeneous, we use \_\_\_\_\_.  
a) CRD  
b) RBD  
c) LSD  
d) All of these
- 3) Smaller the experimental error \_\_\_\_\_ efficient the design.  
a) less  
b) more  
c) not  
d) none of these
- 4) In  $2^2$  factorial experiment with factors A and B, the interaction AB has \_\_\_\_\_ d.f.'s.  
a) 8  
b) 4  
c) 1  
d) depending on the experiment
- 5) In one half fraction with  $I=+ABC$  is called \_\_\_\_\_ fraction.  
a) principal  
b) alternate  
c) complementary  
d) both b and c
- 6) The rank of the incidence matrix in case of BIBD with  $v-1$  treatment in  $b$  block is \_\_\_\_\_.  
a)  $b-1$   
b)  $v-1$   
c)  $v$   
d)  $bv-1$
- 7) The aliased defining relation of  $2^{k-1}$  design is  $I=ABCD$ , then a alias of AB is \_\_\_\_\_.  
a) ACD  
b) BCD  
c) ABD  
d) CD
- 8) The objects which are to be compared in comparative experiment are called \_\_\_\_\_.  
a) treatment  
b) blocks  
c) unit  
d) none of these
- 9) If ABC and BCD are confounded with incomplete block in  $2^n$  experiment, then automatically confounded effect is \_\_\_\_\_.  
a) ABC  
b) AC  
c) AD  
d) B

- 10) For  $2^4$  design the complete model would contain \_\_\_\_\_ effects.
  - a) 16
  - b) 14
  - c) 15
  - d) 32
- 11) BIBD is \_\_\_\_\_ orthogonal.
  - a) Always
  - b) Not
  - c) Sometimes
  - d) All of these
- 12) Preferably \_\_\_\_\_ interactions is chosen for confounding.
  - a) low order
  - b) middle order
  - c) higher order
  - d) none of these
- 13) Confounding is necessary to reduce \_\_\_\_\_.
  - a) Block size
  - b) No. of blocks
  - c) No. of factors
  - d) All of these
- 14) In the design matrix of Randomized block design all entries are \_\_\_\_\_.
  - a) One
  - b) zero
  - c) zero and one
  - d) any value between -1 and +1

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define main effect and interaction effect in factorial design.
- 2) Define Balancedness in design.
- 3) Write down two way ANOVA without interaction model with its assumptions.
- 4) Show that the Randomized Block Design is orthogonal design.
- 5) Write down aliases structure for  $2^{3-1}$  design with generator as a higher order interaction.

**B) Write short notes. (Any Two) 06**

- 1)  $2_{III}^{6-2}$  fractional factorial design
- 2) Complete confounding
- 3) i) Resolution IV  
ii) Resolution V in Design

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Define half fraction of  $2^4$  design with ABCD as g defining generator. Write a alias structure of it.
- 2) Discuss the use of confounding. State and describe the types of confounding.
- 3) Define BIBD. Obtain the determinant of incidence matrix in case of symmetric BIBD.

**B) Answer the following questions. (Any One) 06**

- 1) Describe two way ANOVA without interaction model with one observation per cell and obtain least square estimates of its parameter.
- 2) Define confounding. State its advantages and disadvantages.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Explain  $\frac{1}{4}$ th fraction of  $2^k$  experiment. Construct  $\frac{1}{4}$ th fraction of  $2^6$  design with suitable example.
- 2) Discuss the two way ANOVA without interaction and ANOCOVA in one way case.
- 3) Describe the  $2^3$  factorial experiments. Explain the Yates procedure in case of  $2^3$  designs.

**B) Answer the following questions. (Any One) 04**

- 1) Write down layout of  $2^4$  confounded design in two blocks with higher order interaction is confounded.
- 2) Define
  - i) Principle Fraction
  - ii) Randomization in Design of Experiment

**Q.5 Answer the following questions. (Any Two) 14**

- a) Discuss the basic principles of Design of Experiments.
- b) What are fractional factorial experiments? Illustrate with  $r = 1$  and  $r = 2$  one example.
- c) State difference between analysis of  $2^2$  factorial experiments with  $r = 2$ . Explain full analysis of  $2^2$  factorial experiments for  $r = 1$  and  $r = 2$ .



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**M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**REGRESSION ANALYSIS**

Day & Date: Saturday, 09-11-2019  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) The LSE in general linear model is unique if \_\_\_\_\_.  
 a) coefficient matrix is full rank  
 b) coefficient matrix is non-full rank  
 c) if generalized inverse of coefficient matrix exist  
 d) none of these
- 2) Any vector in estimation space is \_\_\_\_\_ to any vector in error space.  
 a) linear  
 b) orthogonal  
 c) projected  
 d) normalized
- 3) The model  $Y = \beta_0 e^{\beta_1 X} \epsilon$  can be linearized by using \_\_\_\_\_ transformation.  
 a) square root  
 b) reciprocal  
 c) logarithmic  
 d) none of these
- 4) Forward selection procedure begins with the assumption that there are \_\_\_\_\_.  
 a) no regressors in the model  
 b) all regressors in the model  
 c) some regressors in the model  
 d) none of these
- 5) In simple linear regression model  $Y = \beta_0 + \beta_1 X + \epsilon$ ,  $\beta_0$  and  $\beta_1$  are \_\_\_\_\_.  
 a) slope and intercept  
 b) intercept and slope  
 c) error and slope  
 d) intercept and error
- 6) In multiple linear regressions, variance of LSE of  $\beta$  is \_\_\_\_\_.  
 a)  $(X'X)\sigma^2$   
 b)  $(X'X)^{-1}\sigma^2$   
 c)  $X(X'X)^{-1}X'\sigma^2$   
 d)  $\sigma^2$
- 7) The condition number of  $(X'X)$  matrix is given as \_\_\_\_\_.  
 a)  $\lambda_{max} - \lambda_{min}$   
 b)  $\lambda_{max} + \lambda_{min}$   
 c)  $\frac{\lambda_{max}}{\lambda_{min}}$   
 d)  $\frac{\lambda_{min}}{\lambda_{max}}$
- 8) If we use unit length scaling for the regressor variables then  $X'X$  matrix of scaling regressors will be in the form of \_\_\_\_\_.  
 a) covariance matrix  
 b) correlation matrix  
 c) identity matrix  
 d) none of these
- 9) If the coefficient of determination ( $R^2$ ) is near to 1 then it leads to the conclusion that \_\_\_\_\_.  
 a) a good linear relation exists  
 b) there is a lack of linear relationship  
 c) there is a curvilinear relation  
 d) none of these

- 10) The hat matrix  $H = X(X'X)^{-1}X'$  is \_\_\_\_\_.
  - a) symmetric and orthogonal
  - b) symmetric and idempotent
  - c) skew symmetric matrix
  - d) identity matrix
- 11) The multicollinearity in linear regression concerns with \_\_\_\_\_.
  - a) The error terms
  - b) The regressors
  - c) The response variable values
  - d) The coefficient
- 12) The LSE of  $\beta$  for the model  $Y = X\beta + \epsilon$  can be written as \_\_\_\_\_.
  - a)  $\beta + (X'X)^{-1}\epsilon$
  - b)  $\beta + (X'X)\epsilon$
  - c)  $\beta + X'\epsilon$
  - d)  $\beta + (X'X)^{-1}X'\epsilon$
- 13) The regression model  $Y = \beta_0 + \beta_1X + \beta_2X^2$  is called \_\_\_\_\_ model.
  - a) linear
  - b) non-linear
  - c) polynomial
  - d) none of these
- 14) In usual notations,  $var(\hat{Y}) =$  \_\_\_\_\_.
  - a)  $H\sigma^2$
  - b)  $\sigma^2$
  - c)  $(I - H)\sigma^2$
  - d)  $H(I - H)\sigma^2$

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define the coefficient of determination  $R^2$  and adj.  $R^2$ . Derive the relation between them.
- 2) Define  $K^{\text{th}}$  order polynomial regression model in one variable.
- 3) Define condition number and condition indices of  $X'X$  matrix.
- 4) Explain the procedure of computing  $\lambda$ , the parameter of power transformation.
- 5) Define intrinsically model. Give an example.

**B) Write short notes. (Any Two) 06**

- 1) Variance stabilizing transformation
- 2) Prediction interval for the model  $Y = X\beta + \epsilon$
- 3) Cubic spline and cubic-B spline

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Define residual. Obtain its mean and variance.
- 2) With usual notations, prove that  $R^2$  is the square of correlation between  $Y$  and its predicted value  $\hat{Y}$ .
- 3) Show that any solution to normal equations minimizes the residual sum of squares.

**B) Answer the following questions. (Any One) 06**

- 1) Describe cochrane-orkut method for parameter estimation in the presence of autocorrelation.
- 2) Propose an unbiased estimator of error variance  $\sigma^2$  in the regression model and prove your claim.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Describe polynomial models in one variable and two variables.
- 2) Define mallow's  $c_p$  statistic and explain how it is used for variable selection in regression.
- 3) Describe detection of multicollinearity using variance inflation factor.

**B) Answer the following questions. (Any One) 04**

- 1) Define ridge estimator of regression coefficients. Obtain the mean square error of the ridge estimator.

2) Justify whether the following are linear models or not.

i)  $Y = \alpha + \beta X$

ii)  $Y = \alpha \beta \epsilon$

iii)  $Y = \beta_0 + \beta_1 X + \epsilon$

iv)  $Y = \alpha + \frac{\beta}{X} + \epsilon$

Where  $\epsilon \sim iid N(0, \sigma^2)$

**Q.5 Answer the following questions. (Any Two)**

**14**

- 1) State and prove Gauss-Mark off theorem.
- 2) Describe multiple linear regression model stating the assumptions, obtain mean and variance of LSE  $\hat{\beta}$  of  $\beta$ .
- 3) Define non-linear regression model. Discuss least squares method for parameter estimation in non-linear regression.

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**M.Sc. (Semester - IV) (New) (CBCS) Examination Oct/Nov-2019**  
**Statistics**

**DISCRETE DATA ANALYSIS**

Day & Date: Monday, 04-11-2019  
Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory and carry equal marks.  
2) Figures to the right indicate full marks.

**Q.1 Multiple Choice Questions.**

**14**

- 1) Adjacent category model is used for \_\_\_\_\_.  
 a) Nominal response variable      b) Ordinal response variable  
 c) Continuous response variable      d) None of these
- 2) In GLM if response variable has Normal distribution then GLM reduces to \_\_\_\_\_.  
 a) Linear regression model      b) Logistic regression model  
 c) Polytomous regression model      d) None of these
- 3) Which of the following is complementary log-log link function?  
 a)  $\log(-\log(1 - \mu))$       b)  $\log(-\log(1 - \mu))$   
 c)  $\log(1 - \mu)$       d) None of these
- 4) If the response variable is ordinal type, then which of the following model is used \_\_\_\_\_.  
 a) Polytomous logistic regression      b) Cumulative logit model  
 c) Both a and b      d) None of these
- 5) The kernel of the log-likelihood function base on the sample from Poisson distribution is given by \_\_\_\_\_.  
 a)  $\sum_{ijk} x_{ijk} \log m_{ijk}$       b)  $\sum_{ijk} x_{ijk}^2 \log m_{ijk}$   
 c)  $\sum_{ijk} x_{ijk} m_{ijk}$       d)  $\sum_{ijk} m_{ijk} \log x_{ijk}$
- 6) Null distribution of deviance is \_\_\_\_\_.  
 a)  $\chi^2$       b)  $F$   
 c)  $t$       d) None of these
- 7) In logistic regression, response distribution may follow \_\_\_\_\_ distribution.  
 a) Bernoulli      b) Binomial  
 c) Multinomial      d) All the above
- 8) If response variable is count type then, \_\_\_\_\_ is appropriate response Distribution.  
 a) Poisson      b) Negative binomial  
 c) Both a) and b)      d) Neither a) nor b)
- 9) In log linear model for I X J X K table one-factor terms have \_\_\_\_\_ d. f.  
 a) IJK -1      b) (I-1) + (J-1) + (K-1)  
 c) (I-1)(J-1)(K-1)      d) None of these



**Q.5 Answer the following (Any Two)**

- a) What is overdispersion? Write consequences of overdispersion.
- b) Explain residual analysis in GLM.
- c) Write down log-linear model for 2 X J table. Obtain relation between  $U_{12}$  term and cross product ratio.

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**M.Sc. (Semester - IV) (New) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**INDUSTRIAL STATISTICS**

Day & Date: Wednesday, 06-11-2019  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) \_\_\_\_\_ is not a seven SPC tool.
 

a) histogram	b) check sheet
c) single sampling plan	d) pareto chart
- 2) \_\_\_\_\_ is helpful in searching the root-cause of a problem.
 

a) Flow chart	b) Control chart
c) Check sheet	d) Fishbone diagram
- 3) Generally, in process control, cost of production is \_\_\_\_\_ as compared to that in product control.
 

a) high	b) low
c) almost the same	d) exactly the same
- 4) Control chart is \_\_\_\_\_ tool.
 

a) an on-line process control
b) an off-line process control
c) a product control
d) both a process and product control
- 5) \_\_\_\_\_ variability is unavoidable.
 

a) Chance-cause
b) Assignable cause
c) Both chance and assignable cause
d) None of chance and assignable cause
- 6) The probability of type II error for  $\bar{X}$  chart with 3  $\sigma$ -limits and with usual assumptions \_\_\_\_\_.
 

a) is 0.027
b) is 0.9973
c) depends on the size of a shift in the process mean
d) cannot be determined
- 7) Shewhart chart is a particular case of \_\_\_\_\_.
 

a) CUSUM chart
b) EWMA chart
c) Both CUSUM and EWMA charts
d) SPRT chart
- 8)  $C_p$  \_\_\_\_\_  $C_{pk}$ 

a) $\leq$	b) $\geq$
c) $<$	d) $>$

- 9) When  $\mu = \frac{LSL+USL}{2}$ ,
- |                                  |                                  |
|----------------------------------|----------------------------------|
| a) $C_p \leq C_{pk} \leq C_{pm}$ | b) $C_p \geq C_{pk} \geq C_{pm}$ |
| c) $C_p \geq C_{pm} = C_{pk}$    | d) $C_p = C_{pk} = C_{pm}$       |
- 10) \_\_\_\_\_ invented the PDCA cycle.
- |               |           |
|---------------|-----------|
| a) Shewhart   | b) Deming |
| c) Montgomery | d) Fisher |
- 11) The full form of 'M' in DMAIC is\_\_\_\_\_.
- |            |             |
|------------|-------------|
| a) Metric  | b) Material |
| c) Measure | d) Mean     |
- 12) Acceptance sampling is used for all but which one of these?
- |                          |                             |
|--------------------------|-----------------------------|
| a) Incoming raw material | b) Work-in-progress         |
| c) Final goods           | d) Incoming purchased parts |
- 13) In acceptance sampling, the risk of rejecting a good quality lot is known as \_\_\_\_\_.
- |                    |                    |
|--------------------|--------------------|
| a) Consumer's risk | b) Producer's risk |
| c) a Type II error | d) a type I error  |
- 14) The maximum number of defective items that can be found in the sample and still lead to acceptance of the lot is called \_\_\_\_\_.
- |                             |                          |
|-----------------------------|--------------------------|
| a) the upper limit          | b) the acceptance number |
| c) the acceptance criterion | d) AQL                   |

**Q.2 A) Answer the following. (Any Four) 08**

- 1) Define quality from manufacturer's perspective.
- 2) Explain any two dimensions of quality.
- 3) Describe the control statistic of a CUSUM chart for monitoring a downward shift in the process mean.
- 4) Define process capability index.
- 5) What ppm of nonconforming products corresponds to the Six Sigma level when the mean of the key quality characteristic is subject to vary within the middle  $3\sigma$  range of the quality characteristic?

**B) Write Notes. (Any Two) 06**

- 1) Control limits and specifications limits for a quality characteristic.
- 2) V-mask CUSUM procedure.
- 3) Power requirements in designing a sampling inspection plan.

**Q.3 A) Answer the following. (Any Two) 08**

- 1) Describe phase I of control chart.
- 2) Describe c chart.
- 3) Describe double sampling plan.

**B) Answer the following. (Any One) 06**

- 1) Described the DIMAC cycle.
- 2) Explain the construction and operation of an EWMA control chart for monitoring the process mean.

**Q.4 A) Answer the following. (Any Two) 10**

- 1) Describe process control.
- 2) State various sensitizing rules used in control charting.
- 3) Describe an algorithm of obtaining a single attribute sampling plan based on binomial distribution.



**B) Answer the following. (Any One) 04**

- 1) Describe Pareto chart.
- 2) Describe moving average control chart.

**Q.5 Answer the following. (Any two) 14**

- 1) Describe construction, operation, and the underlying statistical principle of  $p$  chart.
- 2) Describe construction, operation, and the underlying statistical principle of Hotelling's  $T^2$  chart.
- 3) Define process capability index  $C_p$  with the necessary underlying assumptions. State and prove its relationship with the probability of nonconformance.



- 11) For which of the following family, each member has non-monotonic failure rate?  
 a) exponential  
 b) log-normal  
 c) Weibull  
 d) Gamma
- 12) Let  $X_{(r)}$  be the  $r^{\text{th}}$  order statistic in a random sample of size  $n$  taken from exponential distribution with mean  $\theta$ . Then  $E[X_{(r)}] = \underline{\hspace{2cm}}$ .  
 a)  $\theta[n - r + 1]^{-1}$   
 b)  $\sum_{i=1}^r [n - i + 1]^{-1}$   
 c)  $\sum_{i=1}^r \theta [n - i + 1]^{-1}$   
 d)  $\frac{1}{\theta} \sum_{i=1}^r [n - i + 1]$
- 13) Which one of the following is not true?  
 a) When there is no censoring K-M estimator is empirical distribution function  
 b) K-M estimator always exists  
 c) K-M estimator is self consistent  
 d) K-M estimator is also known as product moment estimator
- 14) The censoring time for every censored observation is identical in \_\_\_\_\_ censoring.  
 a) type I  
 b) type II  
 c) random  
 d) both in a and b

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) Define IFR and IFRA class of distributions.
- 2) Define associated random variables and state any two properties of associated random variables.
- 3) Give two definitions of star shaped function.
- 4) Describe random censoring with suitable example
- 5) Define Kaplan-Meier estimator.

**B) Write notes. (Any Two) 06**

- 1) Getian's two sampling test under censoring
- 2) Burnham's measure of structural importance
- 3) Star shaped function

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Define minimal path set and minimal cut set.
- 2) Show that if  $F$  is IFR then  $F$  is IFRA
- 3) Discuss maximum likelihood estimation of parameters of Weibull distribution based on a complete sample.

**B) Answer the following questions. (Any One) 06**

- 1) Obtain MLE for mean of exponential distribution under type II censoring.
- 2) For a coherent system with  $n$  components, prove that:
  - i)  $\phi(0)=0$  and  $\phi(1)=1$
  - ii)  $\prod_{i=1}^n x_i \leq \phi(x) \leq \prod_{i=1}^n x_i$

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Define mean residual life function and obtain the same for exponential distribution.
- 2) Obtain the likelihood function under random censoring setup, when the observations come from a distribution  $F$  with density  $f$ .

3) Give two real life examples where both left and right censoring occurs.

**B) Answer the following questions. (Any One) 04**

1) Describe Kaplan-Meier estimator and derive an expression for the same.

2) Define K-out-of-n system. Obtain the reliability function of the system.

**Q.5 Answer the following questions. (Any Two) 14**

a) Explain Mantel's technique of computing Gehan's statistic for a two-sample problem for testing equality of two life distributions.

b) Define mean time to failure (MTTF) and mean residual life (MRL) function. Obtain the same for exponential distribution.

c) Show that IFR class of life distributions is closed under convolution.

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**M.Sc. (Semester - IV) (New) (CBCS) Examination Oct/Nov-2019**  
**Statistics**

**OPTIMIZATION TECHNIQUES**

Day & Date: Monday, 11-11-2019  
Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below. 14**

- 1) Which of the following is not assumption of LPP?
  - a) Certainty
  - b) Additively
  - c) Creativity
  - d) Proportionality
- 2) To maintain optimality of current solution for a change  $\Delta c_k$  in the coefficient  $c_k$  of non basic variable, we must have \_\_\_\_\_.
  - a)  $\Delta c_k = z_k - c_k$
  - b)  $\Delta c_k \geq z_k - c_k$
  - c)  $\Delta c_k \leq z_k - c_k$
  - d)  $\Delta c_k = z_k$
- 3) Slack variable \_\_\_\_\_.
  - a) Which can be added in less than equal to constraint
  - b) Which can be added in greater than equal to constraint
  - c) Which can be a added both types of constraint
  - d) Which can be added in equality type constraint
- 4) Redundant constraint \_\_\_\_\_.
  - a) Can not affect on feasible solution space
  - b) If we add then decrease the feasible solution space
  - c) If we add then increase the feasible solution space
  - d) None of these
- 5) Dual simplex method applicable to those LPP's that starts with \_\_\_\_\_.
  - a) An infeasible solution
  - b) An infeasible but optimum solution
  - c) A feasible solution
  - d) A feasible and optimal solution
- 6) At any iteration of the usual simplex method, if there exist at least one basic variable in the basis at zero level and all  $z_j - c_j \geq 0$ , the current solution is \_\_\_\_\_.
  - a) Infeasible
  - b) Unbounded
  - c) Non-degenerate
  - d) Degenerate
- 7) In mixed integer programming problem \_\_\_\_\_.
  - a) Different objective functions are mixed together
  - b) All of the decision variables requires integer solutions
  - c) Only few of the decision variables requires integer solutions
  - d) None of these
- 8) Branch and bound method divides the feasible solution space into smaller parts by \_\_\_\_\_.
  - a) Enumerating
  - b) Branching
  - c) Bounding
  - d) All of the above

- 9) Dynamic programming deals with the \_\_\_\_\_.
  - a) Multistage decision making problems
  - b) Single stage decision making problems
  - c) Time dependent decision making problems
  - d) Problem which fix the levels of different so as to maximize profit or minimize cost.
- 10) The pay of value for which each player in the game always selects the same strategy is called the \_\_\_\_\_.
  - a) Equilibrium point
  - b) Saddle point
  - c) Both (a) and (b)
  - d) Pivot point
- 11) Recursive approach method used in \_\_\_\_\_.
  - a) Dynamic programming
  - b) Linear programming
  - c) Quadratic programming
  - d) Goal programming
- 12) The of pay-off matrix of a game can be reduced by using the principle of \_\_\_\_\_.
  - a) Dominance
  - b) Inversion
  - c) Transpose
  - d) Rotation reduction
- 13) If the quadratic form  $X^T Q X$  is positive definite, then it is \_\_\_\_\_.
  - a) Strictly convex
  - b) Strictly concave
  - c) Convex
  - d) Concave
- 14) Quadratic programming problem concern with non linear programming problem with quadratic objective function subject to \_\_\_\_\_.
  - a) Non linear inequality constraints
  - b) Non linear equality constraints
  - c) linear inequality constraints
  - d) No constraints

**Q.2 A) Answer the following questions.(Any Four) 08**

- 1) Define general linear programming problem. Also explain the terms solution and feasible solution.
- 2) Explain a dynamic programming problem.
- 3) Describe two persons zero sum game.
- 4) Explain effect of addition of new variable on the optimality of optimum feasible solution.
- 5) Write down characteristics of dynamic programming.

**B) Write Notes.(Any Two) 06**

- 1) Two phase method
- 2) Dominance property
- 3) Non-linear programming problem

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Find the maximum value of  $Z = 50x_1 + 60x_2$ , subject to constraints  $2x_1 + 3x_2 < 1500, 3x_1 + 2x_2 \leq 1500, 0 \leq x_1 \leq 400, 0 \leq x_2 \leq 400$
- 2) Solve the following game with payoff matrix of player A

$$\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix} \end{matrix}$$

- 3) Write down Gomory's fractional cut method to solve all integer programming problem.

**B) Answer the following questions. (Any One) 06**

- 1) Write down simplex algorithm to solve linear programming problem.
- 2) Solve following LPP using dynamic programming  
 $Maximize Z = 3x_1 + 7x_2$ , subject to constraints  $x_1 + 4x_2 < 8, 0 \leq x_2 \leq 2, x_1 \geq 0$

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Explain the terms convex set and convex combinations. Also show that set of all feasible solutions is convex.
- 2) Let  $x_0$  and  $w_0$  be the feasible solutions of primal  $\{Maximize f(x) = cx, sub. to Ax \leq b, x \geq 0\}$  and dual  $\{\min g(w) = b'w, sub to A'w \geq c', w \geq 0\}$  problems respectively. Show that  $x_0$  and  $w_0$  are optimal solutions to the respective problems if and only if  $cx_0 = b'w_0$
- 3) Write an procedure to obtain solution of quadratic programming using Wolfe's method.

**B) Answer the following questions. (Any One) 04**

- 1) Discuss procedure to obtain 2x2 games without saddle point.
- 2) State and prove complementary slackness theorem.

**Q.5 Answer the following questions. (Any Two) 14**

- 1) Use Branch and Bound method to solve following integer programming problem

$Maximize Z = 7x_1 + 9x_2$ , subject to constraints.  
 $-x_1 + 3x_2 < 6, 7x_1 + x_2 \leq 35, x_2 \leq 7, x_1, x_2 \geq 0$  and integers

- 2) Use simplex method to solve following game.

	Player B		
	4	2	4
Player A	2	4	1
	4	1	8

- 3) Describe effect of change in coefficients of objective function  $c_j$ 's in sensitivity analysis.

<b>Seat No.</b>	
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**M.Sc. (Semester - IV) (New) (CBCS) Examination Oct/Nov-2019**  
**Statistics**  
**DATA MINING**

Day & Date: Thursday, 14-11-2019  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 Fill in the blanks by choosing correct alternatives given below.** **14**

- 1) Removing duplicate records is a process called \_\_\_\_\_.  
 a) recovery  
 b) data cleaning  
 c) data washing  
 d) data pruning
- 2) Which of the following is the other name of Data mining?  
 a) Exploratory data analysis.  
 b) Data driven discovery.  
 c) Deductive learning.  
 d) All of the above
- 3) The full form of KDD is \_\_\_\_\_.  
 a) Knowledge database.  
 b) Knowledge discovery in database.  
 c) Knowledge data house.  
 d) Knowledge data definition.
- 4) Task of inferring a model from labeled training data is called \_\_\_\_\_.  
 a) supervised learning  
 b) unsupervised learning  
 c) both (a) and (b)  
 d) none of these
- 5) \_\_\_\_\_ maps data into predefined groups.  
 a) Regression  
 b) Time series analysis  
 c) Prediction  
 d) Classification
- 6) \_\_\_\_\_ is the input to KDD.  
 a) Data  
 b) Information  
 c) Query  
 d) Process
- 7) Treating incorrect or missing data is called as \_\_\_\_\_.  
 a) selection  
 b) preprocessing  
 c) transformation  
 d) interpretation
- 8) \_\_\_\_\_ data are noisy and have many missing attribute values.  
 a) Discretized  
 b) Cleaned  
 c) Real-world  
 d) Transformed
- 9) Market-basket problem was formulated by \_\_\_\_\_.  
 a) Agrawal et al.  
 b) Steve et al.  
 c) Toda et al.  
 d) Simon et. al.
- 10) The absolute number of transactions supporting X in Transactional database is called \_\_\_\_\_.  
 a) confidence  
 b) support  
 c) support count  
 d) none of the above
- 11) The second phase of Apriori algorithm is \_\_\_\_\_.  
 a) Candidate generation  
 b) Itemset generation  
 c) Pruning  
 d) Partitioning



- 12) \_\_\_\_\_ clustering technique starts with as many clusters as there are records, with each cluster having only one record.  
  - a) Agglomerative
  - b) divisive
  - c) Partition
  - d) Numeric
  
- 13) In \_\_\_\_\_ algorithm each cluster is represented by the centre of gravity of the cluster.  
  - a) Factor analysis
  - b) k-means
  - c) STIRR
  - d) ROCK
  
- 14) The sigmoid function also known as \_\_\_\_\_ functions.  
  - a) regression
  - b) logistic
  - c) probability
  - d) neural

**Q.2 A) Answer the following questions. (Any Four) 08**

- 1) What is meant by data mining?
- 2) Define association rule.
- 3) State anti-monotone property.
- 4) Give an example of an activation function.
- 5) Define metadata.

**B) Write Notes. (Any Two) 06**

- 1) What is unidirectional association? Explain with suitable example.
- 2) What are the major tasks in data mining?
- 3) Write a short note on Divisive Hierarchical clustering method.

**Q.3 A) Answer the following questions. (Any Two) 08**

- 1) Explain CRISP data mining process.
- 2) Describe complete linkage method of clustering.
- 3) Write a short note on Outlier Analysis.

**B) Answer the following questions. (Any One) 06**

- 1) Explain McCulloch-Pitts ANN model.
- 2) Describe single layer feed forward network in the context of ANN.

**Q.4 A) Answer the following questions. (Any Two) 10**

- 1) Write a note on grid based clustering method.
- 2) Explain support vector machine in brief.
- 3) Define:
  - i) Accuracy
  - ii) Sensitivity
  - iii) Specificity and
  - iv) Precision, in the context of evaluating classifier performance.

**B) Answer the following questions. (Any One) 04**

- 1) Describe supervised and unsupervised learning.
- 2) Write a note on Market Basket Analysis.

**Q.5 Answer the following questions. (Any Two)**

- 1) Write a note on Density Based Spatial Clustering of Application with Noise (DBSCAN) algorithm.
- 2) Consider the following transactional database D. Assuming minimum support 60% and minimum confidence of 80%, find all frequent items using apriori algorithm. Also give strong association rule.

TID	Items
T100	K,A,D,B
T200	D,A,C,E,B
T300	C,A,B,E
T400	B,A,D

- 3) Explain Naive Baye's classifier.