SLR-JP-309

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
NUMBER THEORY

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. 14

1) For $n > 1, \sum_{d|n} \mu(d) =$ _________.
   a) 0 b) 1
   c) 2 d) 3

2) The largest integer value [e] = _________.
   a) 1 b) 2
   c) 3 d) None of these

3) Euler's theorem states that for any integer a satisfying $\gcd(a, n) = 1$, ________.
   a) $a^n \equiv 1 \ (mod\ n)$ b) $a^{\varphi(n)} \equiv 1 \ (mod\ n)$
   c) $a^n \equiv 1 \ (mod\ \varphi(n))$ d) $a^{\varphi(n)} \equiv 1 \ (mod\ \varphi(n))$

4) $\sigma(24) =$________.
   a) 12 b) 24
   c) 36 d) 60

5) If prime factorization of $n > 1$ has odd number of distinct prime factors then $\mu(n) =$ _________.
   a) 0 b) 1
   c) -1 d) n

6) $lcm\ (a, b) = ab$ iff _______.
   a) $a \nmid b$ b) $b \nmid a$
   c) $\gcd(a, b) = ab$ d) $\gcd(a, b) = 1$

7) Consider a set $\{12x + 24y \mid x, y \in \mathbb{Z}\}$, then the least positive member of the set is ________.
   a) 4 b) 8
   c) 12 d) 24

8) $\varphi(2n) = \varphi(n)$ iff ______.
   a) $\gcd(2, n) = 2$ b) $\gcd(2, n) = n$
   c) $\gcd(2, n) = 1$ d) None of these

9) Which of the following is true?
   a) $\varphi(n)$ is always an even number
   b) $\varphi(n)$ is always an odd number
   c) $\varphi(n)$ is even for infinitely many values of n
   d) $\varphi(n)$ is even only finitely many values of n

10) Solution of $7x \equiv 1\ (mod\ 11)$ is ________.
   a) 2 b) 4
   c) 6 d) 8

11) Number of prime roots of 31 is ________.
   a) 4 b) 8
   c) 30 d) 31

12) Order of 2 modulo of 31 is __________.
   a) 5 b) 4
   c) 3 d) 2

13) If order of a is k modulo n then ________.
   a) $k \mid n$ b) $n \mid k$
   c) $k \mid \varphi(n)$ d) None of these

14) Simultaneous solutions of $x \equiv 2\ (mod\ 5), x \equiv 3\ (mod\ 7)$ is __________.
   a) 15 b) 30
   c) 67 d) 87

Q.2 A) Answer the following questions. (Any Four) 08

1) If $a \mid bc$, with $\gcd(a,b) = 1$, then prove that $a \mid c$.
2) If p_n is the n^{th} prime number, then prove that $p_n \le 2^{2^{n-1}}$.
3) If $n > 1$ and $a, b, c, d$are integers with $a \equiv b\ (mod\ n), c \equiv d\ (mod\ n)$ then prove that $a + c \equiv b + d\ (mod\ n)$.
4) State the theorem for Mobius inversion formula.
5) If n has a primitive roots r and $ind\ a$ denotes the index of a relative to r, then prove the following.

$$ind\ 1\ \equiv 0\ \big(mod\ \varphi(n)\big) and\ ind\ r\ \equiv 1\ \big(mod\ \varphi(n)\big)$$

B) Answer the following questions. (Any Two) 06

1) If p is an odd prime, then prove that there exists a primitive root r of p such that $r^{p-1} \not\equiv 1\ (mod\ p^2)$.
2) If $ca \equiv cb\ (mod\ n)$ and $\gcd(c, n) = d$ then prove that $a \equiv b\ \left(mod\ \frac{n}{d}\right)$.
3) Find the number of zeros in 50!.

Q.3 A) Answer the following questions. (Any Two) 08

1) Prove that the Diophantine equation $ax + by = c$ has a solution iff $d \mid c$; where $c = \gcd(a, b)$. Further prove that iSof x_0, y_0 is any particular solution of this equation, then all other solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t, y = y_0 - \left(\frac{a}{d}\right)t\ ,\quad \text{where t is any integer}$$

2) Find all solutions to the congruence relation $36x \equiv 8\ (mod\ 102)$.
3) If f and F are number-theoretic function such that

$$F(n) = \sum_{d|n} f(d)$$

Then for any positive integer N, prove that

$$\sum_{n=1}^{N} F(n) = \sum_{k=1}^{N} f(k) \left[\frac{N}{k}\right]$$

B) Answer the following questions. (Any One) **06**

1) i) For each integer $n > 2$, prove that $\varphi(n)$ is even.

   ii) If for $n > 1$, prime factorization of n is $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$, then prove that

$$\varphi(n) = \left(p_1^{k_1} - p_1^{k_1-1}\right)\left(p_2^{k_2} - p_2^{k_2-1}\right) \dots \left(p_r^{k_r} - p_r^{k_r-1}\right).$$

2) If p is a prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ and $a_n \not\equiv 0\ (mod\ p)$ is a polynomial of degree $n \geq 1$ with integral coefficients, then show that the congruence $f(x) \equiv 0 (mod\ p)$ has at most n incongruent solutions modulo p.

Q.4 A) Answer the following questions. (Any Two) **10**

1) State and prove Wilson's theorem.
2) Solve: $4x^9 \equiv 7\ (mod\ 13)$
3) Solve the Diophantine equation : $172x + 20y = 1000$.

B) Answer the following questions. (Any One) **04**

1) Given any integers a and b, with $b > 0$, then prove that there exists unique integers q and r satisfying $a = bq + r, 0 \leq r < b$.
2) If all the $n > 2$ terms of the arithmetic progression

   $p, p + d, p + 2d, \dots, p + (n-1)d$

   are prime numbers, then prove that the common difference d is divisible by every prime $q < n$.

Q.5 Answer the following questions. (Any Two) **14**

a) 1) For positive integers a and b, prove that $\gcd(a, b)\ lcm(a, b) = ab$.

2) Let p be an odd prime and let r be a primitive root of p with the property that $r^{p-1} \not\equiv 1\ (mod\ p^2)$, then for each positive integer $k \geq 2$, prove that $r^{p^{k-2}(p-1)} \not\equiv 1\ (mod\ p^k)$.

b) State and prove fundamental theorem of Arithmetic.

c) Define Euler's function. Prove that the function φ is multiplicative.

SLR-JP-310

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
Object Oriented Programming Using C ++

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Which of the following operators could be overloaded?
   a) Size of b) +
   c) += d) : :

2) Which of the following can, not be passed to a function?
   a) Reference variable b) Arrays
   c) Class objects d) Header files

3) In C++ _____ operator is used for dynamic memory allocation.
   a) Scope resolution b) Conditional
   c) New d) Membership access

4) The _____objects have values that can be tested for various error conditions.
   a) Osstream b) Ofstream
   c) stream d) ifstream

5) The member functions of a derived class can directly access only the _____ data.
   a) Private and Protected b) Private and Public
   c) Protected and Public d) Private, Protected and Public

6) _____ binding means that, an object is bound to its function call at compile time.
   a) Late b) Static
   c) Dynamic d) Fixed

7) The pointer to function is known as _____function.
   a) Forward b) Pointer
   c) Callback d) backward

8) The _____ can, not be directly used to access all the members of the derived class.
   a) Void pointers b) Null pointer
   c) this pointer d) base pointer

9) Which of the following is not the member of class?
   a) Static function b) Friend function
   c) const function d) Virtual function

10) Which of the following type of class allows only one object of it to be created?
   a) Virtual class b) Abstract class
   c) Singleton class d) Friend class

11) Which of the following operator is used for input stream?
   a) > b) »
   c) < d) «

12) How many parameters are there in get line function?
   a) 1 b) 2
   c) 2 or 3 d) 3

13) Which symbol is used to create multiple inheritance?
   a) Dot b) Comma
   c) Dollar d) None of the above

14) Which keyword is used to handle the exception?
   a) Try b) Throw
   c) Catch d) None of these

Q.2 A) Answer of the following questions. (Any Four) **08**
1) What is the application of scope resolution operator in C++?
2) How does a C++ structure differ from a C++ class?
3) Describe the importance of destructor.
4) How Polymorphism is achieved at run time in C++?
5) Explain a pointer to derived class.

B) Write Notes. (Any Two) **06**
1) Friend class
2) Put ( ) and Get ( )
3) Enumerated data types in C++

Q.3 A) Answer of the following questions. (Any Two) **08**
1) What is friend function? What are the merits and demerits of using friend function?
2) Write a program to illustrate how pointers to a derived object are used.
3) Explain this pointer with example.

B) Answer of the following questions. (Any One) **06**
1) What is operator overloading? Why is it necessary to overload an operator?
2) Write a program to demonstrate how a static data is accessed by a static member function.

Q.4 A) Answer of the following questions. (Any Two) **10**
1) What is a virtual base class? Explain with an example.
2) Write a program to show how the binary operator is overloaded using friend function.
3) Explain function template with example.

B) Answer of the following questions. (Any One) **04**
1) How do we invoke a constructor function?
2) What is an exception? How exception is handled in C++.

Q.5 Answer of the following questions. (Any Two) **14**
a) What is Virtual function? Explain rules for virtual functions.
b) What is meant by C++ stream classes? Explain C++ stream classes.
c) Distinguish between overloaded functions and function templates

SLR-JP-311

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
ALGEBRA – I

Day & Date: Tuesday, 05-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Consider the following statements.
P : Every normal series is subnormal
Q : Every composition series is normal series.
Then,
a) P is true but Q is false b) P is false but Q is true
c) Both P and Q are true d) Both P and Q are false

2) Which of the following is true in a commutative ring with unit R?
a) Every maximal ideal is prime b) R is an integral domain
c) R has no zero divisors d) Every prime ideal is maximal

3) $< 2\mathbb{Z}, +, . >$ is not an integral domain because _________.
a) it has zero divisors b) it has unit elements
c) it has no unity d) None of these

4) If G is a group then which of the following necessarily imply that $G^{'} = \{e\}$.
a) G is non-abelian b) G is abelian
c) G is infinite d) All of the above

5) If a group G is infinite cyclic group, then number of generators of G is __________.
a) 0 b) 1
c) 2 d) Infinite

6) Class equation of D_4 is __________.
a) 4=1+1+1+1 b) 8=2+2+2+2
c) 8=1+1+2+2+2 d) 8=1+3+4

7) Which of the following is true?
a) $x^2 + 1$ is irreducible over $\mathbb{Z}_2$ but not over $\mathbb{Z}_3$
b) $x^2 + 1$ is irreducible over $\mathbb{Z}_3$ but not over $\mathbb{Z}_2$
c) $x^2 + 1$ is reducible over $\mathbb{Z}_2$ as well as $\mathbb{Z}_3$
d) $x^2 + 1$ is irreducible over $\mathbb{Z}_2$ as well as $\mathbb{Z}_3$

8) Which of the following is a prime ideal $\mathbb{Z}[x]$ but not maximal?
a) $< x >$ b) $< x, 2 >$
c) $< x, 3 >$ d) $< x, 5 >$

9) Which of the following is a maximal ideal in $\mathbb{Z}$?
a) $< 4 >$ b) $< 6 >$
c) $< 5 >$ d) $< 51 >$

10) Which of the following quotient rings form a field?
a) $\frac{\mathbb{Z}[x]}{< x >}$ b) $\frac{\mathbb{Z}[x]}{< x^2 + 1 >}$
c) $\frac{\mathbb{Z}[x, y]}{< x, y, 5 >}$ d) $\frac{\mathbb{Z}[x]}{< x^2 - 2 >}$

11) If D is a Unique Factorization Domain, then __________.
a) $D[x]$ is UFD b) $D[x]$ need not be a UFD
c) $D[x]$ is ED d) $D[x]$ is PID

12) If D is not Unique Factorization Domain then _________.
a) D is not PID b) D is not ED
c) $D[x]$ is not UFD d) All of the above

13) If $\mathbb{F}$ is a field, then _________.
a) $\mathbb{F}[x]$ is a field b) $\mathbb{F}[x]$ is not an Integral Domain
c) $\mathbb{F}[x]$ is not UFD d) $\mathbb{F}[x]$ is never a field

14) In $\mathbb{Z}[x]$, content of $4x^2 + 6x - 8$ is ________.
a) 1 b) -1
c) 2 d) -2

Q.2 A) Answer the following questions. (Any Four) 08
1) Define Euclidean domain.
2) Prove that S_n is not solvable for $n \geq 5$.
3) Let G be a group G' be its commutator subgroup. Then prove that $\frac{G}{G'}$ is abelian.
4) Show that an element $a \in \mathbb{F}$ is a zero of $f(x) \in \mathbb{F}[x]$ iff $x - a$ is a factor of $f(x)$ in $\mathbb{F}[x]$.
5) In a Principal Ideal Domain, prove that if an irreducible $p \mid ab$, then either $p \mid a$ or $p \mid b$.

B) Answer the following questions. (Any Two) 06
1) Show by an example that a subnormal series need not be normal.
2) Show that no group of order $p^r, r > 1$ is simple.
3) For a Euclidean domain with Euclidean valuation v, prove that $v(1)$ is minimal among all $v(a)$ for nonzero $a \in D, and\ u \in D$ is a unit iff v(u)=1.

Q.3 A) Answer the following questions. (Any Two) 08
1) Prove that a group G is solvable iff there exists a positive integer K such that $G^{(k)} = \{e\}$.
2) If G is a finite group of order p^n, p prime and if X is a G-set then prove that $|X| \equiv |X_G| (mod\ p)$.
3) Prove that an ideal $< p >$ in a PID is maximal iff p is an irreducible.

B) Answer the following questions. (Any One) 06
1) If G is a finite group then prove that any two Sylow-p-subgroupss of G are conjugates of each other.
2) State and prove Eisenstein's criteria.

Q.4 A) Answer the following questions. (Any Two) 10
1) If D is a UFD and if $\mathbb{F}$ is a field of quotients of D. If $f(x)(\deg(f(x)) > 0)$ is an irreducible in $D[x]$, then prove that $f(x)$ is also in irreducible in $\mathbb{F}[x]$. Also, if $f(x)$ is a primitive in $D[x]$ and irreducible in $\mathbb{F}[x]$, then prove that $f(x)$ is irreducible in $\mathbb{F}[x]$.
2) Prove that homomorphic image of a nilpotent group is nilpotent.

3) State and prove Burnside theorem.

B) Answer the following questions. (Any One) **04**

1) Prove that no group of order 48 is simple.
2) If $\mathbb{F}$ is a field, then prove that $\mathbb{F}[x]$ is Unique Factorization Domain.

Q.5 Answer the following questions. (Any Two) **14**

a) State and prove Sylow's first theorem

b) Find the isomorphic refinements of the following series.

$$\{0\} < 245\mathbb{Z} < 49\mathbb{Z} < Z \; and \; \{0\} < 60\mathbb{Z} < 20\mathbb{Z} < Z$$

c) If D is Unique Factorization Domain, then prove that $D[x]$ is also Unique Factorization Domain.

SLR-JP-312

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
REAL ANALYSIS – I

Day & Date: Thursday, 07-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Second derivative of f measures __________.
   a) Convexity of f b) Concavity of f
   c) Both a and b d) None of these

2) The supremum of set of all lower sums is called __________.
   a) Upper integral b) Lower integral
   c) Both a and b d) None of these

3) If $f : S \to T$ is called open mapping if for every open set A in S, the image $f(A)$ is _________.
   a) Closed b) open in T
   c) open in S d) Closed in T

4) Oscillatory sum of f is given by __________.
   a) $M_i - m_i$ b) $M - m$
   c) $w(f, p)$ d) $w(P, f)$

5) The value of integral always lies between _______ & ______ of function corresponding to any partition.
   a) $M(b - a)$ & $m(a - b)$ b) $m(a - b)$ & $M(a - b)$
   c) $m(b - a)$ & $M(b - a)$ d) None of these

6) For every partition P, which of the following is true?
   a) $\mu \leq \Delta\, x_i$ b) $\mu \geq \Delta\, x_i$
   c) both a and b d) none

7) The total derivative of _________ function is function itself.
   a) Non-Linear b) Linear
   c) Continuous d) Differentiable

8) If $F: \mathbb{R}^n \to \mathbb{R}^m$ $(m \neq 1)$ then Total derivative is ________.
   a) real number b) Gradient vector
   c) real matrix d) None of these

9) i) If directional derivative is exist in every direction then partial derivative exist.
   ii) If all partial derivative exist then directional derivatives exist.
   a) Only (i) is true b) only (ii) is true
   c) both (i) and (ii) are false d) both (i) and (ii) are true

10) The Set $L(x, y)$ is the set of all points on the line segment joining x and y in $\mathbb{R}^n$ is convex set if _______.
   a) $L(x, y) \supset S$ b) $L(x, y) \supseteq S$
   c) $L(x, y) \subseteq S$ d) None of these

11) The maxima of $f(x) = x^3 - 12x + 4$ in $[-3, 3]$ is __________.
   a) -12 b) 18
   c) 20 d) 24

12) Which of the following is not convex sets?
   a) Circle b) Sphere
   c) Cone d) Line segment

13) The minima of $f(x) = x^3 - 27x + 4$ in $[-3, 3]$ is ________.
   a) 50 b) 18
   c) -51 d) -50

14) The jacobian determinant is given by _________.
   a) $J_f(x)$ b) $|D_j f_i(x)|$
   c) $\frac{\partial(f_1,f_2,\dots,f_n)}{\partial(x_1,x_2,\dots,x_n)}$ d) All (a), (b) and (c) correct

Q.2 A) Answer the following questions. (Any Four) **08**

1) If f be bounded and integrable function on $[a, b]$ and $f(x) \geq 0;\ \forall\, x\ \in [a, b]$ then prove that
$$\int_a^b f\,dx\ \geq 0 \qquad ;\ a\ \leq b$$
$$\int_a^b f\,dx\ \geq 0 \qquad ;\ a\ \geq b$$
2) State second form of condition form of integrability.
3) State Darboux's condition of integrability with respect to α.
4) Show that the function $f^{'}(c)(v)$ is a bounded function.
5) If $f(z) = u + iv$ is a complex valued function with derivative at each point z in $\mathbb{C}$ then prove that $J_f(z) = |\ f^{'}(z)|^2$.

B) Answer the following questions. (Any Two) **06**

1) If f & g be bounded and integrable on $[a, b]$ such that $f \geq g$ on $[a, b]$ then prove that
$$\int_a^b f\,dx\ \geq\ \int_a^b g\,dx$$
2) If f be bounded with respect to α on $[a, b]$ such that for every for every $\varepsilon > 0$ there is a partition P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$ then prove that f is integrable on $[a, b]$ with respect to α.
3) Explain the geometrical interpretation of Directional derivative of function.

Q.3 A) Answer the following questions. (Any Two) **08**

1) If f & g be bounded and integrable on $[a, b]$ and if g keeps same sign on $[a, b]$ then prove that there is a real number μ lies between the bounds of f such that
$$\int_a^b (fg)dx = \mu \int_a^b g\,dx$$
2) i) Define Darboux's condition of integrability of bounded function f with respect to α. **01**
   ii) If f be a bounded function on $[a, b]$, prove that for any partition $P_1\ and\ P_2$ of $[a, b]$ **03**
$$L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$$
3) i) Define Total derivative. **01**
   ii) Whether the Mean Value Theorem for the function $f: \mathbb{R} \to \mathbb{R}$ holds if f is vector valued? **03**

B) Answer the following questions. (Any One) **06**

1) Prove that the oscillation of bounded function f on $[a, b]$ is the supremum of the Set $\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a, b]\}$

2) i) Show that the quantity $f^{'}(c)h$ is a linear function. **02**

ii) If f be a differentiable function at a point $'c'$ with total derivative T_c then prove that the directional derivative exist at a point $'c'$ for every $'u'$ in $\mathbb{R}^n$ and $T_c(u) = f^{'}(c; u)$ **04**

Q.4 A) Answer the following questions. (Any Two) **10**

1) If f is non-negative continuous function on $[a, b]$ such that $\int_a^b f\, dx = 0$ then prove that $f(x) = 0;\ \forall\, x \in [a, b]$

2) If $f = (f_1, f_2, \ldots, f_n)$ has continuous partial derivative $D_j f_i$ on open set S in $\mathbb{R}^m$ and $j_f(a) \neq 0$ for some $'a'$ in S then prove that there is an n-ball $B(a)$ on which f is one-one.

3) If $f: S \to \mathbb{R}^m$ be a mapping where $S \subseteq \mathbb{R}^n$. If $v = v_1u_1 + v_2u_2 + \cdots + v_nu_n$ where $u_1, u_2, \ldots, u_n$ be unit co-ordinate vectors in S. then prove that $f^{'}(c)(v) = \sum_{k=1}^{n} v_k D_k f(c)$.
Inpaticular if $m = 1$ then prove that $f^{'}(c)(v) = \nabla f(c). v$, where this is the dot product of $\nabla f(c)$ with v, $\nabla f(c) = \big(D_1 f(c), D_2 f(c), \ldots., D_n\, f(c)\big)$.

B) Answer the following questions. (Any One) **04**

1) If $\int_a^b f\, dx$ & $\int_a^b\, g\, dx$ exist and if f is monotone on $[a, b]$ then prove that there exist $\xi \in [a, b]$ such that $\int_a^b (f\ g)dx = f(a)\int_a^{\xi} g\, dx + f(b)\int_{\xi}^b g\, dx$.

2) Verify that the Directional derivative at a point $O = (0,0)$ exist or not in the direction of $u = (u_1, u_2)$.

i) **02**
$$f(x, y) = \frac{xy}{x^2 + y^2} \quad ; (x, y) \neq 0$$
$$= 0 \quad ; (x, y) = 0$$

ii) **02**
$$f(x, y) = \frac{xy}{x + y} \quad ; (x, y) \neq 0$$
$$= 0 \quad ; (x, y) = 0$$

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that : A necessary and sufficient condition for the integrability of a bounded f is that $lim_{\mu(P)\to 0}\big(U(P, f) - L(P, f)\big) = 0$

b) If P^* is refinement of P then prove that

i) $U(P^*, f, \alpha) \le U(P, f, \alpha)$

ii) $L(P^*, f, \alpha) \ge L(P, f, \alpha)$

c) If f and all its partial derivatives of order less than $'m'$ are differentiable at each point of an open set S in $\mathbb{R}^n$ and a and b are two points of S such that $L(a, b) \subseteq S$, then prove that there is a point $'z'$ on the line segment $L(a, b)$ such that

$$f(b) - f(a) = \sum_{k=1}^{m-1} \frac{f^{(k)}(a; b-a)}{k!} + \frac{f^{(m)}(z; b-a)}{m!}$$

SLR-JP-313

Seat No.	

Set **P**

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
DIFFERENTIAL EQUATIONS

Day & Date: Saturday, 09-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If r is a root of multiplicity m of a polynomial p, $\deg(p) \geq 1$ then _____.
   a) $p^{(m)}(r) \neq 0$ b) $p^{(m)}(r) = 0$
   c) $p^{(m)}(r) < 0$ d) none of these

2) Two functions x, |x| are linearly _____.
   a) linearly independent b) linearly dependent
   c) both a and b d) none of these

3) Initial value problem for second order differential equation is denoted by _____.
   a) $L(y) = 0, y(x_0) = 0, y'(x_0) = 0$ b) $L(y) = 0$
   c) $L(y) = 0, y(x_0) = \alpha, y'(x_0) = \beta$ d) none of these

4) The Bessels equation is _____.
   a) $x^2y" + xy' + (x^2 - \alpha^2)y = 0$ b) $x^2y" + xy' + (x^2 - \alpha^2)y = 1$
   c) $x^2y" - xy' + (x^2 - \alpha^2)y = 0$ d) none of these

5) The solutions of $y" - 4y = 0$ are
   a) sin2x, cos2x b) sinx, cosx
   c) 2x, -2x d) none of these

6) Three functions ϕ_1, ϕ_2, ϕ_3 are said to linearly independent if _____.
   a) $W(\emptyset_1, \emptyset_2, \emptyset_3)(x) = 0$ b) $c_1 = 0, c_2 = 0, c_3 = 0$
   c) $c_1\emptyset_1 + c_2\emptyset_2 + c_3\emptyset_3 = 0$ d) none of these

7) Singular point of differential equation is same as regular singular point.
   a) true b) false
   c) may or may not d) none of these

8) Initial value problem $y' = f(x, y), y(x_0) = 0$ has _____ solution.
   a) infinitely many b) unique
   c) two d) none of these

9) The regular singular point of $xy" + 4y = 0$ is _____.
   a) 1 b) -1
   c) 0 d) no regular singular point

10) The function g is analytic at x_0 if g can be expressed in power series about x_0 which has _____ radius of convergence.
   a) positive b) negative
   c) zero d) none

11) The function $\sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$ is

a) Bessel function of zero order of second kind
b) Bessel function of zero order of first kind
c) Bessel function of order α of first kind
d) Bessel function of order 3 of first kind

12) If p is polynomial such that $\deg(p) = n$ and $p(z) = (z - a)q(z)$ then q has _____ root.

a) n	b) n-1
c) n+1	d) o

13) If p is polynomial such that $\deg(p) \geq 1$ then p has _____ root.

a) at list one	b) at most one
c) more than two	d) none of these

14) If $\propto \pm\beta i$ are two complex conjugate roots of characteristic equation then two solutions are given by

a) $cos\beta x, sin\alpha x$	b) $e^{\alpha x} cos\beta x, e^{\alpha x} sin\beta x$
c) $e^{\alpha x}, e^{\beta x}$	d) none of these

Q.2 A) Answer the following questions. (Any Four) **08**

1) Solve $y''' = x^2$
2) Find the general solution of $y'' + 4ky' - 12k^2y = 0$
3) Show that the functions cosx, sinx are linearly independent for $-\infty < x < \infty$.
4) Write indicial polynomial for n^{th} order Euler equation.
5) Write solution of $y' + ay = b(x)$ Where a is constant and b is continuous function.

B) Write Notes. (Any Two) **06**

1) Define singular point with example.
2) Solve $y'' + 5y' + 6y = 0$
3) Show that $f(x, y) = 4x^2 + y^2$ satisfy Lipschitz condition on $S: |x| \leq 1, |y| \leq 1$

Q.3 A) Answer the following questions. (Any Two) **08**

1) Suppose ϕ_1, ϕ_2 are linearly independent solutions of the constant coefficient equation $y^{(2)} + a_1y^{(1)} + a_2y = 0$. Show that W is a constant if $a_1 = 0$.
2) Find the singular point of $3x^2y'' + x^6y' + 2xy = 0$, is it regular singular.
3) Compute the Wronskian of the solutions of LDE $y''' - 4y' = 0$

B) Answer the following questions. (Any One) **06**

1) Prove the Uniqueness theorem for homogenous second order differential equation.
2) Solve $y' = xy, y(0) = 1$ using the method of successive approximation.

Q.4 A) Answer the following questions. (Any Two) **10**

1) Prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)}W(\phi_1, \phi_2)(x_0)$ if ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing point x_0.
2) Find the solution of $y'' - 2y' - 3y = 0, y(0) = 0, y'(0) = 1$
3) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if $W(\phi_1, \phi_2)(x) \neq 0$

B) Answer the following questions. (Any One) **04**

1) Prove the Existence theorem for homogenous second order differential equation
2) Find the three cube roots of 4i

Q.5 Answer the following questions. (Any Two) **14**

1) Derive Bessel function of zero order of the first kind.

2) Let $\phi_1 \neq 0$ be a solution of $L(y) = 0$ on an interval I. If $v_1, v_2, \dots, v_n$ is any basis on I for the solution of
$\phi_1 v^{(n-1)} + \cdots + \left[n\phi_1{}^{(n-1)} + (n-1)\phi_1{}^{(n-2)} + .. + a_{n-1}\phi_1\right]v = 0$ of order $n - 1$ and if $v_k = u'_k$ (k=2, 3,….n) then prove that $\phi_1, u_2\phi_1, \dots u_n\phi_1$ is a basis for the solution of $L(y) = 0$ on I.

3) Prove that basis for the solution of Euler equation on any interval not containing $x = 0$ is $|x|^{r_1}$ and $|x|^{r_2}$ in case r_1, r_2 are distinct roots of $q(r)$.

SLR-JP-314

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination Oct/Nov-2019
Mathematics
CLASSICAL MECHANICS

Day & Date: Wednesday, 13-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If determinant of a matrix A is ± 1 then A is ________
 a) Orthogonal matrix b) Invertible matrix
 c) Non-orthogonal matrix d) Both b and c

2) Hamiltonian (H) is a function independent of _________.
 a) Generalized coordinates b) Generalized velocities
 c) Generalized momentum d) Time

3) If $A = [a_{ij}]_{4\times 4}$ represents rotation matrix then it's degree of freedom are _____.
 a) 16 b) 8
 c) 4 d) 6

4) Gravitational force is an example of _________.
 a) Conservative force b) Non-conservative force
 c) Both a and b d) Neither a nor b

5) The conjugate momentum P_k for generalized coordinate q_k is ________.
 a) $\frac{\partial L}{\partial q_k}$ b) $\frac{\partial L}{\partial P_k}$
 c) $\frac{\partial L}{\partial \dot{q}_k}$ d) $\frac{\partial L}{\partial \dot{P}_k}$

6) If q_k is cyclic in Lagrangian L then P_k represents _________.
 a) Constant energy b) Non constant energy
 c) Constant motion d) Non constant motion

7) Number of Cartesian coordinates to describe configuration of double pendulum is/are _________.
 a) One b) Two
 c) Three d) Four

8) The extremum of the functional $J[y(x)]$ is called local maximum if _______.
 a) $\Delta J \leq 0$ b) $\Delta J \geq 0$
 c) $\Delta J = 0$ d) $\Delta J > 0$

9) $\int_{t_1}^{t_2} (L + H) dt$ is ________.
 a) Energy b) Action
 c) Brachistochrone d) Momentum

10) Routhian is a function which usually replaces _________.
 a) Lagrangian b) Hamiltonian
 c) Both a and b d) Neither a nor b

11) A string of length l moving in the plane then its degrees of freedom are _________.
 a) 4 b) 3
 c) 2 d) 1

12) Number of generalized coordinates for describing Atwood's machine is/are __________.
 a) 1 b) 2
 c) 3 d) 4

13) Shortest distance between two points is a __________.
 a) Circle b) Parabola
 c) Catenary d) Straight line

14) If A is an orthogonal matrix then __________.
 a) $A^{-1} = A$ b) $A^2 = A$
 c) $A.A^T = I$ d) $A^{-1}.A^T = I$

Q.2 A) Answer the following questions. (Any Four) **08**

1) State principle of least action.
2) State fundamental lemma of calculus of variations.
3) Define generalized coordinates.
4) Define constraints.
5) Define Hamiltonian.

B) Write Notes. (Any Two) **06**

1) Eulerian angles
2) Routhian
3) Physical significance of Hamiltonian

Q.3 A) Answer the following questions. (Any Two) **08**

1) Set up Lagrangian for Atwood's machine.
2) If q is cyclic in L then show that q is cyclic in H.
3) Explain law of conservation of energy and momentum.

B) Answer the following questions. (Any One) **06**

1) Find equations of motion of compound pendulum using Hamiltonian formulation.
2) Find necessary condition for the extremum for

$$I = \int_a^b F\,(x, y, y', y'')dx$$

Q.4 A) Answer the following questions. (Any Two) **10**

1) Find the extremal of the functional $\int_0^1 [(y')^2 + 12xy]dx$ subject to $y(0) = 0, \quad y(1) = 1$
2) Find Lagrange's equations of motion of a simple pendulum.
3) Find kinetic energy of a particle of mass M on the surface of earth.

B) Answer the following questions. (Any One) **04**

1) Derive an expression of generalized velocity.
2) Show that two successive orthogonal Transformation of orthogonal matrix is also orthogonal.

Q.5 Answer the following questions. (Any Two) **14**

a) Derive Euler's equation for the motion of a rigid body with one point fixed.

b) Derive Lagranges equations of motion in terms of Kinetic energy from D'Alembert's principle.

c) Using Hamiltonian formulation, find equations of motion of a dynamical system whose Lagrangian is $L = \frac{\dot{x}^2}{2} - \frac{\omega^2 x^2}{2} - \propto x^3 + \beta\, x\, \dot{x}^2$ where α, β are constants.

SLR-JP-315

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Mathematics
ALGEBRA – II

Day & Date: Monday, 04-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If F is a finite field of q elements and $F \subseteq K$ is also a finite field such that $[K:F] = n$ then K has _________ elements.
   a) q^2 b) q^n
   c) n^q d) n^2

2) The splitting field of $x^2 - 1$ over Q is _________.
   a) Q(i) b) R
   c) Q d) C

3) If a and b are algebraic over F of degrees m and n respectively then $\frac{a}{b}(b \neq 0)$ is algebraic of degree _________ over F.
   a) atmost mn b) equal to mn
   c) atmost m/n d) equal to m/n

4) I) A set of rational numbers is a subfield of R.
   II) A set of irrational numbers is a subfield of R.
   a) Only I is true b) Only II is true
   c) Both are true d) Both are false

5) A field K is regarded as a vector space over _________ of K.
   a) any subset b) any subfield
   c) any subring d) any subgroup

6) Every complex number is algebraic over _________.
   a) Q b) R
   c) Q' d) None

7) A field C of complex numbers is __________ extension of field R of real numbers.
   a) finite b) simple
   c) algebraic d) All of these

8) If $[K:F] = m$ then each element in K is algebraic over F of degree ______.
   a) equal to m b) less than m
   c) greater than m d) atmost m

9) If a and b are algebraic over F of degrees m and n respectively and m, n are relatively prime then F(a,b) is of degree ________ over F.
   a) mn b) at most mn
   c) at least mn d) m+n

10) I) The number $2^{1/4}$ is constructible.
    II) The number cos120 is constructible.
    a) Only I is true b) Only II is true
    c) Both are true d) Both are false

11) If characteristic of $F = p \neq 0$ and $f'(x) = 0$ for $f(x) \in F[x]$ then $f(x) =$ ______
    a) constant b) zero
    c) $g(x), g(x) \in F[x]$ d) $g(x^p), g(x) \in F[x]$

12) The number of automorphisms on a field of real numbers is/are ________.
    a) 1 b) 0
    c) 2 d) Finite

13) The extension $Q(\sqrt{5}, \sqrt{11})$ is a _________ extension of Q.
    a) finite b) algebraic
    c) separable d) All of these

14) Every finite extension is a simple extension. This statement is true for a field of characteristic __________.
    a) finite b) prime
    c) nonprime d) zero

Q.2 A) Answer the following (Any Four) **08**

1) Prove that: Every finite extension is algebraic extension.
2) Find degree and basis of $Q(2^{1/4}, i)$ over Q.
3) Check whether $\sqrt{5 - \sqrt{11}}$ is algebraic over Q or not.
4) If a and b are constructible numbers then prove that $a/b(b \neq 0)$ is also constructible.
5) Find all automorphisms on $Q(\sqrt{17})$.

B) Write Notes on (Any Two) **06**

1) Galois group
2) Splitting field
3) Algebraic element and its degree

Q.3 A) Answer the following (Any two) **08**

1) If K is an extension of a field F, $a \in K$ is algebraic over F and a satisfies an irreducible polynomial $p(x)$ for a over F then prove that $p(x)$ must be minimal polynomial for a over F.
2) If $f(x) \in F[x]$ be a polynomial of degree greater or equal to 1 with α as a root then prove that α is a multiple root iff $f'(\alpha) = 0$.
3) Prove that: Any two splitting field of the same polynomial over a given field are isomorphic by an isomorphism leaving every element of F fixed.

B) Answer the following (Any One) **06**

1) Prove that: A polynomial can have at most n roots in any extension field.
2) If K is finite extension of a field F of characteristic zero, H is subgroup of $G(K, F)$ and K_H is fixed field of H then prove that $[K: K_H] = o(H)$.

Q.4 A) Answer the following (Any Two) **10**

1) If $p(x)$ is an irreducible polynomial in $F[x]$ of degree $n \geq 1$ then prove that there is an extension E of F such that $[E: F] = n$ in which $p(x)$ has a root.
2) If ψ be an isomorphism of a field F onto a field F' defined by $\psi(\alpha) = \alpha'$ for every $\alpha \in F$, corresponding to a polynomial $f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \cdots + \alpha_n x^n$ in $F[x]$ there is a polynomial $f'(t) = \alpha_0' + \alpha_1' t + \alpha_2' t^2 + \cdots + \alpha_n' t^n$ in $F'[t]$ then prove that the splitting fields E and E' of $f(x)$ in $f[x]$ and $f'(t)$ in $F'[t]$ respectively are isomorphic by an isomorphism ϕ with a property that $\phi(\alpha) = \psi(\alpha) = \alpha'$ for every $\alpha \in F$.
3) Find splitting field of $x^4 - 2$ over Q.

B) Answer the following (Any One) **04**

1) Find the Galois group of $x^2 - 2$ over field of rational numbers.
2) If α be zero of a polynomial $p(x) = x^2 + x + 1 \epsilon Z_2[x]$ is irreducible over Z_2 then find $Z_2(\alpha)$ and its addition and multiplication tables.

Q.5 Answer the following (Any two) **14**

a) Prove that : A field K is normal extension of a field F of characteristic zero iff K is splitting field of some polynomial over F.

b) Prove that: Any two finite fields having the same number of elements are isomorphic.

c) If $f(x)$ in $f[x]$ is irreducible over f then prove that all roots of $f(x)$ have same multiplicity.

SLR-JP-316

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Mathematics
REAL ANALYSIS – II

Day & Date: Wednesday, 06-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If ϕ is an empty set, then $m^*(\phi) =$ ________.
   a) ϕ b) A
   c) I_n d) 0

2) Every Borel set is ________.
   a) Measurable set b) Non measurable set
   c) Empty set d) Dense set

3) If χ_A is a characteristic function of a set A, then $\chi_{A\cap B} =$ _________.
   a) χ_A b) χ_B
   c) $\chi_A\chi_B$ d) $\chi_A + \chi_B$

4) A nonnegative measurable function f is called integrable over the measurable set E, if _________.
   a) $\int_E f = \infty$ b) $\int_E f < \infty$
   c) $\int_E f > \infty$ d) $\int_E f < E$

5) The positive part f^+ of a function f is given by $f^+(x) =$________.
   a) $\max\{f(x), 0\}$ b) $\max\{-f(x), 0\}$
   c) $\min\{f(x), 0\}$ d) $f(x)$

6) Fatou's lemma remains valid if 'convergence a.e.' is replaced by _______.
   a) convergence in measure b) convergence
   c) does not converge d) convergence in integral

7) If f is a function then $D_-f(x) =$ __________.
   a) $\overline{\lim_{h\to 0^+}} \dfrac{f(x+h) - f(x)}{h}$ b) $\overline{\lim_{h\to 0^+}} \dfrac{f(x) - f(x-h)}{h}$
   c) $\underline{\lim_{h\to 0^+}} \dfrac{f(x+h) - f(x)}{h}$ d) $\underline{\lim_{h\to 0^+}} \dfrac{f(x) - f(x-h)}{h}$

8) If f is a real-valued function defined on the interval $[a, b]$ and if $T_a^b(f) < \infty$, then we say that f is _________.
   a) of bounded variation over $[a, b]$
   b) not of bounded variation over $[a, b]$
   c) of negative variation over $[a, b]$
   d) of positive variation over $[a, b]$

9) Sum of two absolutely continuous functions __________.
   a) need not be absolutely continuous
   b) is absolutely continuous
   c) is singular
   d) need not be continuous

10) A function φ is concave if $-\varphi$ is __________.
   a) convex
   b) concave
   c) semi-concave
   d) strictly convex

11) If f is an integrable function on $[a, b]$, then its definite integral to be the function F is defined on $[a, b]$, by _________.
   a) $F(x) = \int_a^x F(t)dt$
   b) $F(x) = \int_a^x f(t)dt$
   c) $F(x) = \int_a^b F(t)dt$
   d) $F(x) = \int_a^b f(t)dt$

12) If f is a function of bounded variation defined on $[a, b]$ and if $a \leq c \leq b$, then $T_a^c(f) + T_c^b(f) =$ _________.
   a) $T_b^a(f)$
   b) $T_a^b(f)$
   c) $T_a^c(f)$
   d) $T_c^b(f)$

13) If $D^+f(x) = D_+f(x)$, then we say that f has _________.
   a) right-hand derivative at x
   b) derivative at x
   c) left-hand derivative at x
   d) right-hand derivative at 0

14) If φ and ψ are simple functions which vanish outside a set of finite measure and if $\varphi \geq \psi$, then ________.
   a) $\int \varphi \leq \int \psi$
   b) $\int \varphi < \int \psi$
   c) $\int \varphi \geq \int \psi$
   d) $\int \varphi > \int \psi$

Q.2 A) Answer the following questions. (Any Four) **08**

1) State Egoroff's theorem.
2) Define convex function.
3) Define outer measure $m^*(A)$ of the set A.
4) State Bounded Convergence Theorem.
5) Define absolutely continuous function f on $[a, b]$.

B) Write Notes. (Any Two) **06**

1) If f and g are two measurable real-valued functions defined on same domain, then prove that $f + g$ is also measurable.
2) Show that $D^+[-f(x)] = -D_+f(x)$.
3) If f is integrable over E, then show that $|f|$ is also integrable and
$$|\int_E f| \leq \int_E |f|$$

Q.3 A) Answer the following questions. (Any Two) **08**

1) Show that for given any set A and any $\epsilon > 0$, there is an open set O such that $A \subseteq O$ and $m^*(O) \leq m^*(A) + \epsilon$.
2) If E_1 and E_2 are measurable sets, then show that
$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$$
3) If f is integrable function over E, and if A and B are disjoint measurable sets contained in E, then prove that
$$\int_{A \cup B} f = \int_A f + \int_B f$$

B) Answer the following questions. (Any One) **06**

1) If f is integrable on $[a, b]$, then prove that the function F defined by
$$F(x) = \int_a^x f(t)dt$$
is a continuous function of bounded variation on $[a, b]$.

2) If $E \subseteq [0, 1)$ is a measurable set, then prove that for each $y \in [0, 1)$ the set $E \dot{+} y$ is measurable and $m(E \dot{+} y) = m(E)$.

Q.4 A) Answer the following questions. (Any Two) **10**

1) If f is nonnegative function, and $\langle E_i \rangle$ is a disjoint sequence of measurable sets, and if $E = \bigcup_{i=1}^{\infty} E_i$ then prove that
$$\int_E f = \sum_{i=1}^{\infty} \int_{E_i} f$$

2) If E is a given set, then prove that the following statements are equivalent:
   $\alpha)$ E is measurable.
   $\beta)$ Given $\epsilon > 0$, there is an open set $O \supset E$ such that $m^*(O \sim E) < \epsilon$.
   $\gamma)$ There is a G in G_δ with $E \subset G$, $m^*(G \sim E) = 0$.

3) If φ is a continuous function on (a, b) and if $D^+\varphi$ is nondecreasing, then prove that φ is convex function.

B) Answer the following questions. (Any One) **04**

1) If $m^*(E) = 0$, then prove that E is a measurable set.

2) If f is a function defined by
$$f(x) = \begin{cases} 0 & if\ x = 0, \\ x \sin\dfrac{1}{x} & if\ x \neq 0, \end{cases}$$
then find $D_+ f(0)$.

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that a function f is of bounded variation on $[a, b]$, if and only if f is the difference of two monotone real-valued functions on $[a, b]$.

b) Prove that the outer measure of an interval is its length.

c) If f is a bounded and measurable function on $[a, b]$ and if
$$F(x) = \int_a^x f(t)dt + F(a),$$
then prove that $F'(x) = f(x)$ for almost all $x \in |a, b|$.

SLR-JP-317

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Mathematics
GENERAL TOPOLOGY

Day & Date: Friday, 08-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Every closed subset of a compact space is ________.
   a) Compact b) Open
   c) Never compact d) None of these

2) Every singleton set in __________ T-space is open.
   a) compact b) discrete
   c) indiscrete d) both discrete and indiscrete

3) If $< X, \beth >$ is a discrete topological space with X, an uncountable set. Then $< X, \beth >$ is _________.
   a) Lindelof
   b) Second axiom space
   c) Not a second axiom space
   d) Both Lindelof and Second axiom space

4) Which of the following is true in a T-space $< X, \beth >$, for $A, B \subseteq X$.
   a) $d(A \cup B) = d(A) \cup d(B)$ b) $d(A \cap B) = d(A) \cap d(B)$
   c) $d(A) \cap d(B) \subseteq d(A \cap B)$ d) $i(A \cup B) \subseteq i(A) \cup i(B)$

5) If B is a closed set in $< X, \beth >$ with $A \subseteq X$ such that every closed set containing A contains B, then
   a) $A = B$ b) $\bar{B} = A$
   c) $\bar{B} = \bar{A}$ d) No such set B exists

6) If $f : < X, \beth > \to < X^*, \beth^* >$ is a function and $a \in X^*$ is a fixed element. If $f(x) = a \forall x \in X$, then
   a) f is continuous at some point of X only
   b) f is continuous at a
   c) f is not continuous at X
   d) f is continuous at X

7) In a T-space $< X, \beth >$, if A, B are two non-empty disjoint subsets of X such that A and B has no limit points in common then A and B are called as _________.
   a) connected sets b) separated sets
   c) compact sets d) bounded sets

8) A completely regular T_1-space is known as ________.
   a) T_1 space b) T_2 space
   c) T_3 space d) $T_{\frac{3}{2}}$ space

9) Being a normal space is __________.
   a) not a topological property b) hereditary property
   c) closed hereditary property d) absolute property

10) Compact subset of a T_2 space is __________.
   a) open b) closed
   c) clo-open d) connected

11) Every T_1 space need not be _________.
   a) T_2 b) T_0
   c) Both T_0 and T_2 d) Compact

12) Let X be an uncountable set and $p \in X$. Then p-exclusion topology on X is ________.
   a) Separable b) Not Separable
   c) Not compact d) None of these

13) If B is any closed set containing A in $< X, \beth >$ then ________.
   a) $d(A) \subseteq A$ b) $d(A) \subseteq B$
   c) $d(A) = B$ d) $B \subseteq d(A)$

14) A subset A of $< X, \beth >$ is said to be open if __________.
   a) $i(A) = A$ b) $c(A) = A$
   c) $b(A) = A$ d) $e(A) = \emptyset$

Q.2 A) Answer the following questions. (Any Four) **08**

1) Define : Closure of a set, continuity of a function in a topological space
2) Let $\beth$ and $\beth^*$ be any two topological on $X(\neq \emptyset)$ with $\mathfrak{B}$ and $\mathfrak{B}^*$ as bases. If each $G \in \beth$ is union of members of $\mathfrak{B}^*$, then prove that $\beth \leq \beth^*$.
3) Prove that being a locally compact space is a topological property,
4) Prove that $< R, \beth_u >$ is a first axiom space.
5) Let $< X, \beth >$ be any topological space and let $< Y, \beth^* >$ be a T_2-space. Let f and g be continuous mappings of X into Y. If f and g agree on a dense subset of X. Then prove that $f = g$ on the whole X.

B) Answer the following questions. (Any Two) **06**

1) Prove that continuous image of a connected space is a connected space.
2) Prove that every second axiom space is a separable space.
3) Let $< X, \beth >$ be a T_1-space and let $A \subseteq X$. If a point $x \in X$ is a limit point of A, then prove that any open set (neighbourhood) containing x contains infinitely many points of A.

Q.3 A) Answer the following questions.(Any Two) **08**

1) Let $< X, \beth >$ and $< Y, \beth^* >$ be any two topological spaces. Let $< X, \beth >$ be a compact space and let $f : X \to Y$ be an onto continuous map. Then prove that $< Y, \beth^* >$ is compact.
2) Prove that a T-space is a Hausdorff space iff any two disjoint compact subsets of X can be separated by disjoint open sets.
3) Prove that every second axiom space is a first axiom space. Is converse true? Justify your answer.

B) Answer the following questions.(Any One) **06**

1) Let $< X, \beth >$ and $< X^*, \beth^* >$ be two topological spaces. Let $f : X \to X^*$ be a bijective mapping. Then prove that following are equivalent.
   a) f is a homeomorphism
   b) f and f^{-1} are continuous
   c) f is a continuous and closed mapping

2) a) Let $< X, \beth >$ be a regular space. Let A and B be disjoint subsets of X such that A is closed and B is compact in X. Then prove that there exist two disjoint open sets in X one containing A and the other containing B.

b) In any topological space $< X, \beth >$, prove that $x \in d(A) \Rightarrow x \in d(A - \{x\}), \forall A \subseteq X$.

Q.4 A) Answer the following questions.(Any Two) **10**

1) Prove that a topological space $< X, \beth >$ is normal iff for any closed set F and an open set G containing F, there exists an open set H such that $F \subseteq H \subseteq \overline{H} \subseteq G$
2) Let $< X, \beth >$ be a first axiom space and $< Y, \beth^* >$ be any topological space. A function $f: X \to Y$ is continuous on X iff f is sequentially continuous.
3) Show that continuous image of a Lindelof space is Lindelof space.

B) Answer the following questions.(Any One) **04**

1) Let $< X, \beth >$ be a completely regular space. Let N be neighbourhood of $x \in X$. Then prove that there exists a continuous function $f: X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1 \forall y \in X - N$ and conversely.
2) Let $< X, \beth >$ be a topological space. If a connected set C has a non-empty intersection with both E and the complement of E in $< X, \beth >$, then prove that C has a non-empty intersection with the boundary of E.

Q.5 Answer the following questions.(Any Two) **14**

a) Let $< X, \beth >$ be a topological space and let A and B be non-empty subsets of X. Then prove that the following are equivalent. 07

1) $X = A|B$
2) $X = A \cup B, \bar{A} \cap \bar{B} = \emptyset$
3) $X = A \cup B, A \cap B = \emptyset$ and A, B are both closed in X
4) $B = X - A$ and A is both open and closed in X
5) $B = X - A$ and $b(A) = \emptyset$
6) $X = A \cup B, A \cap B = \emptyset$, and A, B both are open in X

b) 1) Prove that a topological space $< X, \beth >$ is regular iff for any point $x \in X$ and any open set G contaning x, there exists an open set H such that $x \in H$ and $\overline{H} \subseteq G$. 04

2) Prove that being a T_1-space is a hereditary property. 03

c) 1) Prove that the property of being a second axiom space is a topological property. 04

2) Prove that a co-finite topological space is compact 03

SLR-JP-318

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Mathematics
COMPLEX ANALYSIS

Day & Date: Monday, 11-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If a path $\gamma: [0,1] \to \mathbb{C}$ is rectifiable if ________.
   a) γ is closed
   b) γ is a function of bounded variations
   c) γ is smooth
   d) γ is not closed

2) $\int_{|z|=2} \frac{(z-3)^2}{(z-1)^2} dz =$ ______.
   a) $4\pi i$ b) $-4\pi i$
   c) $8\pi i$ d) $-8\pi i$

3) $\int_{|z|=1} \frac{sinz}{z^2} dz =$
   a) 0 b) πi
   c) $-\pi i$ d) $2\pi i$

4) Every entire function has singularity only at _______.
   a) 0 b) 1
   c) ∞ d) -1

5) If $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ is Laurent series expansion, then f has a pole of order m if _________.
   a) f has m terms in the principal part
   b) f has $(m-1)$ terms in the principal part
   c) f has $(m+1)$ terms in the principal part
   d) f has no terms in the principal part

6) Which of the following conditions implies that f is analyatic?
   a) $f: G \to \mathbb{C}$ is continuous
   b) $f: G \to \mathbb{C}$ is differentiable
   c) $f: G \to \mathbb{C}$ is differentiable and G is open
   d) $f: G \to \mathbb{C}$ is bounded

7) An analytic function f has a pole at $z = a$ if $\lim_{n\to\infty} |f(z)| =$
   a) 1 b) 2
   c) 0 d) ∞

8) If f is analytic then which of the following is analytic?
   a) $f(\bar{z})$ b) $\overline{f(z)}$
   c) $\overline{f(\bar{z})}$ d) None of these

9) $\int_{|z-1|=1} sinz\, dz =$
   a) 0 b) 1
   c) $2\pi i \sin(1)$ d) $2\pi i$

10) Order of zero at $z = 0$ for the function $f(z) = z^3(1 - cos2z)$ is ______.
   a) 1 b) 3
   c) 7 d) 5

11) $Res\left(\frac{sinz}{z^4}, z = 0\right) =$ ______.
   a) $-\frac{1}{6}$ b) $\frac{1}{6}$
   c) 6 d) -6

12) If f is an analytic function on G and $f(z) = 0$ on a closed curve γ in G, then ______.
   a) f cannot be xero b) $f(z) = 0$ on G
   c) $f(z) \neq 0$ on G d) No such function exists

13) Which of the following is not entire?
   a) $f(z) = \sin z$ b) $f(z) = e^z$
   c) $f(z) = \sin(\mathrm{e}^{-z})$ d) $\tan z$

14) $\int_{|z|=1.5} \frac{z}{z-2} dz =$ ______.
   a) 1.5 b) 2
   c) $2\pi i$ d) 0

Q.2 A) Attempt any four of the following questions. 08

1) Define:
   i) Möbius transformation
   ii) Analytic function
2) Find radius of convergence of the series $\sum_{n=0}^{\infty}(3 + 4i)^n z^n$
3) Define Meromorphic function and explain with an example.
4) Evaluate: $\int_\gamma \frac{e^{iz}}{z^2}\, dz$, where $\gamma(t) = re^{it}, 0 \leq t \leq 2\pi$.
5) State Riemann mapping theorem.

B) Attempt any two of the following questions. 06

1) Prove that an isolated singularity of a function f at $z = a$ is a removable singularity iff f is bounded in the neighbourhood of $z = a$.
2) Let G be a region and suppose that f is a non-constant analytic function on G. Then for any open set U in G, prove that $f(U)$ is open.
3) Calculate residue of $f(z) = \frac{z^2}{(z-1)(z+2)^2}$. Find $Res(f, 1)$ and $Res(f, -2)$.

Q.3 A) Attempt any two of the following questions. 08

1) If f has an essential singularity at $z = a$, then prove that for every $\delta > 0, \overline{f\left(ann(a\,; 0, \delta)\right)} = \mathbb{C}$, i.e. $f\left(ann(a\,; 0, \delta)\right)$ is dense in $\mathbb{C}$.
2) If $p(z)$ is a non-constant polynomial then prove that there is a complex number a with $p(a) = 0$.
3) State and prove Rouche's theorem.

B) Attempt any one of the following questions. 06

1) State and prove Schwarz's lemma.
2) Let $f : G \to \mathbb{C}$ be analytic and suppose $\bar{B}\,(a, r) \subseteq G (r > 0)$. if $\gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$, then prove that $f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(w)}{w-z}\, dw$, for $|w - z| < r$.

Q.4 A) Attempt any two of the following questions. 10

1) Let $f : G \to \mathbb{C}$ is an analytic function with $Re\, f(z) \geq 0$ for all z in $D = \{z \,||z| < 1\}$ and $f(0) = 1$, then show that $Re\left(f(z)\right) > 0$ and $\frac{1-|z|}{1+|z|} \leq |f(z)| \leq \frac{1+|z|}{1-|z|},\ z \in D$

2) Evaluate : $\int_0^{\pi} \frac{1}{a+cos\theta} d\theta, a > 1$.

3) Let G be a region and let $f: G \to \mathbb{C}$ be a continuous function such that $\int_T f = 0$ for very triangular path T in G, then prove that f is analytic in G.

B) Attempt any one of the following questions. **04**

1) If G is open and connected and $f: G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G, then prove that f is constant.

2) Let G be a region and $f: G \to \mathbb{C}$ be an analytic function such that there is a point a in G with $|f(a)| \geq |f(z)|$ for all z in G, then prove that f is constant.

Q.5 Attempt any two of the following questions. **14**

a) Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence $R > 0$. Then prove that

1) For each $k \geq 1$, the series $\sum_{n=k}^{\infty} n(n-1)(n-2) \dots (n-k+1) a_n (z-a)^{n-k}$ has radius of convergence R.

2) The function f is infinitely differentiable on $B(a, R)$ and furthermore, $f^{(k)}(z)$ is given by the series $\sum_{n=k}^{\infty} n(n-1)(n-2) \dots (n-k+1) a_n (z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| < R$.

3) For $n \geq 0, a_n = \frac{1}{n!} f^{(n)}(a)$.

b) Let G be a connected open set and let $f: G \to \mathbb{C}$ be an analytic function. Then prove that following are equivalent.

1) $f \equiv 0$

2) $\{z \in G \mid f(z) = 0\}$ has limit point in G

3) There is a point a in G such that $f^{(n)}(a) = 0$ for each $n \geq 0$.

c) Let $D = \{z \mid |z| < 1\} = B(0,1)$ be a unit disc and $\partial D = \{z \mid |z| = 1\}$. Fix $a \in \mathbb{C}$ such that $|a| < 1$. Define the Möbius transformation

$$f_a(z) = \frac{z-a}{1-\bar{a}z}$$

Then prove that

1) f_a is a one-one map of D onto itself

2) f_a is analytic in an open disc containing the closure of D

3) $f_a^{-1} = f_{-a}$

4) f_a maps ∂D onto ∂D.

5) $f_a(a) = 0, f'_a(0) = 1 - |a|^2, f'_a(a) = \frac{1}{1-|a|^2}$

SLR-JP-319

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Mathematics
RELATIVISTIC MECHANICS

Day & Date: Thursday, 14-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Two events are said to be simultaneous if they happen at _________ & ________.
 a) Same time but not necessarily same place
 b) Same place but not necessarily same time
 c) Same time and same place
 d) None of these

2) Newton's laws of motion are ________ under Galian transformation.
 a) Invariant b) Non invariant
 c) Conserved d) Not Conserved

3) The Einstein's time dilation equation is ________.
 a) $dt = \sqrt{1 - v^2/c^2}\, d\tau$ b) $dt' = \sqrt{1 - v^2/c^2}\, d\tau$
 c) $d\tau = \sqrt{1 - v^2/c^2}\, dt$ d) $d\tau' = \sqrt{1 - v^2/c^2}\, dt$

4) Simultaneity of an event is _______ concept.
 a) Absolute b) Invariant
 c) Non absolute d) Non invariant

5) If $v <<< c$ then $u' \oplus v =$ ________.
 a) u' b) $u' + v$
 c) c d) v

6) The classical Doppler effect is ________.
 a) $\gamma = \gamma'(1 - \beta)$ b) $\gamma = \gamma'(1 + \beta)$
 c) $\gamma' = \gamma(1 + \beta)$ d) No Doppler effect

7) Consider the constant ratio $\frac{v_1 - v_2}{u_1 - u_2} = -e$ Then the collision is elastic if ____.
 a) $e = 0$ b) $e = -1$
 c) $e = 1$ d) $e > 0$

8) The transformation equations for mass is given by ________.
 a) $m = \gamma m' \left(1 - u_x' \cdot \frac{v}{c^2}\right)$ b) $m' = \gamma m \left(1 - ux \frac{v}{c^2}\right)$
 c) $m = \gamma m' \left(1 + u_x' \frac{v}{c^2}\right)$ d) None of these

9) If A_{lm} is symmetric and B^{lm} is skew symmetric tensor then $A_{lm}.B^{lm} =$_____.
 a) 1 b) -1
 c) 0 d) Finite

10) The velocity of fluid is _______ but not the acceleration.
   a) Covariant vector b) Contravariant vector
   c) Metric tensor d) Ricci tensor

11) In the process of contraction the rank of tensor is reduced by _______.
   a) 1 b) 2
   c) 3 d) 4

12) Minkowski's space time is _______ but not ________.
   a) Euclidean, flat b) flat, Euclidean
   c) flat, geodesic d) Geodesic, flat

13) The relativistic addition of two velocities u' and v is defined as $u' \oplus v =$ ____.
   a) $\frac{u'-v}{1-\frac{u'v}{c^2}}$ b) $\frac{u'+v}{1-\frac{u'v}{c^2}}$
   c) $\frac{u'+v}{1+\frac{u'v}{c^2}}$ d) $\frac{u'-v}{1+\frac{u'v}{c^2}}$

14) An index which is _______ in single term is called real index.
   a) Repeated b) Not Repeated
   c) Used d) Not used

Q.2 A) Answer the following questions. (Any Four) 08

1) Define
   i) Inertial frame
   ii) Co-incident events
2) Write Lorentz transformation equations.
3) Doppler effect – explain.
4) Write transformation equation of charge density and current density.
5) A body has dimension represented by $5i + 3j$ in s' - frame. How these dimension will be represented in s - frame if s' - is moving with velocity 0.8C along $x - x'$ axis.

B) Write Notes. (Any Two) 06

1) Galian transformation
2) Relativistic Mass
3) Minkowski space time

Q.3 A) Answer the following questions. (Any Two) 08

1) Show that for small velocities the Lorentz transformation reduces to Galian transformation equations.
2) The length of a rocket ship is 100 meters on the ground. When it is in flight its length observed on the ground is 99 meters. Calculate its speed.
3) Obtain an expression for charge density.

B) Answer the following questions. (Any One) 06

1) State postulates of special theory of relativity and deduce the Laurentz transformation equations.
2) Prove or disprove: The electromagnetic wave equation is invariant under Galian transformation.

Q.4 A) Answer the following questions. (Any Two) **10**

1) Prove that: If the rod moves with velocity V relative to the observer then its measured length is contracted in the direction of motion by the factor $\sqrt{1 - v^2/c^2}$.
2) Obtain the relativistic aberration formula from the velocity transformation equations.
3) Derive transformation equation for mass.

B) Answer the following questions. (Any One) **04**

1) Prove that the fourth component of four momentum is energy.
2) Prove that the Kronecker delta symbol is a mixed tensor of rank 2.

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that: Newton's laws of motion are invariant under Galian transformations.
b) Derive an expression for relativistic kinetic energy.
c) Derive Einstein's mass-energy relation.

Seat No.	

Set P

M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Mathematics
FUNCTIONAL ANALYSIS

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. 14

1) The conjugate space of l_1^n is __________.
   a) l_∞^n b) l_1^n
   c) l_2 d) l_1

2) If M is a complete subspace of a normal linear space N and if N/M is a Banach space then N is _________.
   a) Inner product space b) Hilbert space
   c) Banach space d) Sobolev space

3) The normed linear space of all operators on N is denoted by ________.
   a) N b) N^*
   c) $\mathfrak{B}(N^*)$ d) $\mathfrak{B}(N)$

4) If H is a complex inner product space, then for all $x, y, z \in H$ and $\alpha, \beta \in \mathbb{C}$, we have $(x, \alpha y + \beta z) =$ _________.
   a) $\alpha(x, y) + \beta(x, z)$ b) $\bar{\alpha}(x, y) + \bar{\beta}(x, z)$
   c) $\bar{\alpha}(x, y) + \bar{\beta}(y, z)$ d) $\beta(x, y) + \alpha(x, z)$

5) If $\mathbb{R}^n$ is a Banach space under the norm $\|x\| = \max\{|x_1|, |x_2|, \dots, |x_n|\}$ is usually denoted by _________.
   a) l_1^n b) l_2^n
   c) l_∞^n d) l_p^n

6) Each reflexive space is ________.
   a) Banach space b) Hilbert space
   c) Inner product space d) Bessel's space

7) If N and N' are normal linear spaces, and if $T : N \to N'$, then the graph of T is $G_T =$_________.
   a) $\{(x, Tx): x \in N'\}$ b) $\{(x, Tx): x \in N\}$
   c) $\{(x, Tx): x \in T\}$ d) ϕ

8) Complete inner product space is known as __________.
   a) Banach space b) Normed linear space
   c) Hilbert space d) Bessel's space

9) If M and N are two non-empty subspaces of a Hilbert space H and $M \perp N$ then _________.
   a) $M \cap N = \{0\}$ b) $M \cap N = \phi$
   c) $M + N = H$ d) $M \cup N = \{0\}$

10) If $\{e_i\}$ is a complete orthonormal set in a Hilbert space and x is an arbitrary vector in H, then the expansion $x = \sum(x, e_i)e_i$ is called _________.
   a) Fourier expansion of x
   b) Fourier coefficient of x
   c) Bessel's equation
   d) Parseval's equation

11) If T is an operator on a Hilbert space H and M is a closed subspace of H, then M is invariant under T if _________.
   a) $M \subseteq T(M)$
   b) $T(M) = \phi$
   c) $T(M) \subseteq M$
   d) $M = \phi$

12) If N is a non-zero normed linear space then the space N is a Banach space if and only if the set $\{x : \|x\| = 1\}$ is ________.
   a) dense
   b) bound of N
   c) complete
   d) compliment of N

13) Any linear transformation between finite-dimensional spaces is always _________.
   a) continuous
   b) discontinuous
   c) unbounded
   d) semi-continuous

14) The space $C[a, b]$ of all real valued continuous functions defined on $[a, b]$ is __________.
   a) infinite dimensional Banach space
   b) finite dimensional Banach space
   c) infinite dimensional Hilbert space
   d) None of these

Q.2 A) Answer the following questions. (Any Four) **08**

1) If a one-to-one linear transformation T of a Banach space onto itself is continuous, then prove that it inverse T^{-1} is continuous.
2) If x and y are any two orthogonal vectors in a Hilbert space H, then prove that
$$\|x - y\|^2 = \|x\|^2 + \|y\|^2$$
3) If H is a Hilbert space, then show that $H^{\perp} = \{0\}$.
4) If N and N' are normed linear space and T is a linear transformation of N into N', then prove that T is continuous if and only if T is continuous at the origin, in the sense that $x_n \to 0 \Longrightarrow T(x_n) \to 0$.
5) If H is a Hilbert space, and if $T \to T^*$ is the adjoint operation on $\mathfrak{B}(H)$, then prove that $(\mathrm{T}_1\mathrm{T}_2)^* = \mathrm{T}_2^*\mathrm{T}_1^*$.

B) Write Notes. (Any Two) **06**

1) Hahn-Banach theorem
2) Equivalent norms
3) Self-adjoint opetrator

Q.3 A) Answer the following questions. (Any Two) **08**

1) If N is a normal operator on a Hilbert space H, then prove that $\|N^2\| = \|N\|^2$.
2) Describe the projection on linear space L geometrically.
3) If S is a non-empty subset of a Hilbert space, then show that $S \cap S^{\perp} = \{0\}$.

B) Answer the following questions. (Any One) **06**

1) Prove that the mapping $\phi : H \to H^*$ defined by $\phi(y) = f_y$, where $f_y(x) = (x, y)$ for every $x \in H$ is an additive, one-to-one, onto and isometry but one linear.
2) If M is closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exists a functional f_0 in N^* such that $f_0(M)$ and $f_0(x_0) \neq 0$.

Q.4 A) Answer the following questions. (Any Two) **10**

1) If P and Q are the projections on closed linear subspace M and N of a Hilbert space H, then prove that $M \perp N$ if and only if $PQ = 0$ if and only if $QP = 0$.
2) If T is an operator on Hilbert space H for which $(Tx,\ x) = 0$ for all $x \in H$, then prove that $T = 0$.
3) If x and y are two vectors in a Hilbert space H, the prove that
$$|(x, y)| \leq \|x\|\|y\|.$$

B) Answer the following questions. (Any One) **04**

1) If N and $N^{'}$ are normed linear spaces, then prove that $T : N \to N^{'}$ is bounded if and only if T is continuous.
2) Show that an orthonormal set in a Hilbert space *H* is linearly independent.

Q.5 Answer the following questions. (Any Two) **14**

a) If T is an operator on a Hilbert space H, then prove that T is normal if and only if its real and imaginary parts commute.

b) If B and $B^{'}$ are Banach spaces and T is a linear transformation of B into $B^{'}$, then prove that T is continuous if and only if its graph is closed.

c) State and prove Banach fixed point theorem.

SLR-JP-321

Seat No.	

Set P

M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Mathematics
ADVANCED DISCRETE MATHEMATICS

Day & Date: Tuesday, 05-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) A self-complemented distributive lattice is called ________.
   a) Boolean Algebra b) Modular lattice
   c) Boolean lattice d) Complete lattice

2) The sum of coefficient in the expansion of $(w+x+y+z)^5$ is ________.
   a) 3^5 b) 2^5
   c) 4^5 d) 5^5

3) The maximum number of zero element and unit element in a poset is _____.
   a) 0 b) 1
   c) 2 d) None

4) G be a graph with 3 connected components and 24 edges then maximum possible number of vertices is ________.
   a) 27 b) 21
   c) 20 d) None

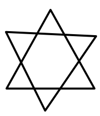
5) In a lattice L, if $a, b \in L$ and $if\ a \wedge b = a$ then __________.
   a) $a \preccurlyeq b$ b) $a \succcurlyeq b$
   c) $a = b$ d) None

6) The expansion of $\dfrac{1}{(1-x)^n} =$ __________.
   a) $\sum_{r=0}^{\infty} n-1+rC_r x^r (-1)^r$ b) $\sum_{r=0}^{\infty} n-1+rC_r x^r$
   c) $\sum_{r=0}^{\infty} (-1)^r n-1+rC_r a^r x^r$ d) None

7) A lattice $(L, \preccurlyeq)$ is _______ iff $(a \wedge b) \vee (a \wedge c) = a \wedge \big(b \vee (a \wedge c)\big) \forall a, b, c \in L$
   a) Modular b) Distributive
   c) Complete d) None

8) A complete bipartite graph Km,n is regular iff __________.
   a) $m > n$ b) $m < n$
   c) $m = n$ d) $m \neq n$

9) The number of connected components of the graph is



   a) 1 b) 2
   c) 3 d) 4

10) The complete graph K_5 has ________ different spanning trees.
   a) 130 b) 110
   c) 120 d) 125

11) In a lattice L if $a \preccurlyeq b$ and $c \preccurlyeq d$ then _________.
   a) $b \preccurlyeq c$ b) $a \preccurlyeq d$
   c) $a \vee c \preccurlyeq b \vee d$ d) None

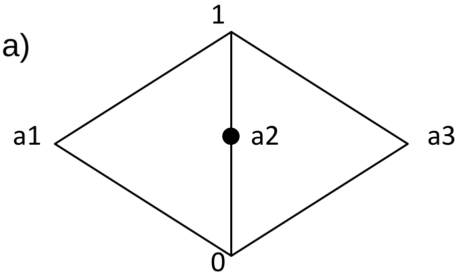
12) $(\{1,2,5,6,10,15,a\},/)$ is lattice if the smallest value of a is ________.
   a) 150 b) 100
   c) 75 d) 30

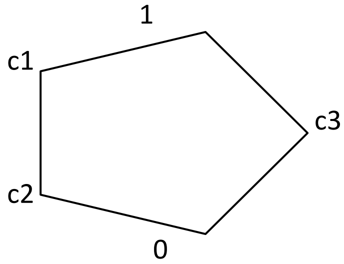
13) There are only _________ non-isomorphic simple graphs on 4 vertices.
   a) 9 b) 10
   c) 12 d) 11

14) A vertex of degree one is called __________.
   a) Isolated b) Pedant
   c) Even d) None

Q.2 A) Answer the following (Any Four) **08**

1) Show that the lattices given in the following figure are no distributive.

a)


b)


2) Explain Handshaking lemma.
3) Draw the graphs having the following matrices as their adjacency matrices.

i) $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$

4) Prove that in a tree addition of any new edge creates exactly one circuit.
5) Find the number of primes less than 100 by using principle of inclusion exclusion.

B) Write Notes. (Any Two) **06**

1) Isomorphism of graphs
2) Regular graph
3) Principal of Inclusion-Exclusion

Q.3 A) Answer the following questions. (Any Two) **08**

1) Show that every chain is distributive lattice.
2) Show that if a graph G contains exactly two odd degree vertices then there is a path between these two vertices.
3) Show that a connected graph with n vertices has atleast (n-1) edges.

B) Answer the following questions. (Any One) **06**

1) Prove that the product of two lattices is a lattice.
2) Let G be a simple graph with n vertices and let $\bar{G}$ be its complement
   i) Prove that for each vertex v in G, $d_G(v) + d_{\bar{G}}(v) = n - 1$
   ii) Suppose that G has exactly one even vertex. Find how many odd vertices does $\bar{G}$ have?

Q.4 A) Answer the following questions. (Any Two) **10**

1) If G be a graph with n vertices and q edges, w(G) denotes the number of connected components of G then show that G has at least n-w(G) edges.
2) Show that in any of five integers from 1 to 8 are choosen then at least two of them will have sum equals to 9.
3) If G be graph with n vertices and A denote the adjacency matrix of G and k be any positive integer then prove that (i.j)th entry of A^K is the number of different v_i-v_j walks in G of length k.

B) Answer the following questions. (Any One) **04**

1) Among the integers 1 to 300 find how many are not divisible by 7 but divisible by 3.
2) Find all the spanning trees of complex graph K_4

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that an edge e of a graph G is a bridge iff e is not a part of any cycle in G.

b) Show that in any lattice the distributive inequalities holds:
1) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$
2) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

c) If L is any lattice then state and prove idempotent law, commutative law, associative law and absorption law.

SLR-JP-322

Seat No.	

Set P

M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Mathematics
LINEAR ALGEBRA

Day & Date: Thursday, 07-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If V and W are inner product spaces over the same field, and T is a linear transformation from V into W, then T preserves distances if for all $\alpha, \beta;\ \|T(\alpha) - T(\beta)\| =$ __________.
   a) 0
   b) 1
   c) $\|\alpha - \beta\|$
   d) None of these

2) If V is a finite dimensional inner product space, T is a linear operator on V and T is invertible, then $(T^*)^{-1} =$ _________.
   a) (T^*)
   b) $(T^{-1})^*$
   c) (T^{-1})
   d) $(T^*)^{-1}$

3) A complex $n \times n$ matrix A is called __________ if $A^* A = AA^*$.
   a) Normal
   b) Self-adjoint
   c) Unitary
   d) Identify

4) A linear operator T on a finite dimensional vector space $V(F)$ is diagonalizable if V is the direct sum of the _________.
   a) eigen vector of T
   b) eigenspaces of T
   c) eigen values of T
   d) minimal and characteristic polynomial

5) If $W_1, W_2, \dots, W_n$ are subspace of a vector space $V(F)$, then $V = W_1 \oplus W_2 \oplus \dots \oplus W_n$ if $V = \sum_{i=1}^{n} W_i$ and ________.
   a) $\sum_{i=1, i\neq j}^{n} W_i = \{0\}$ for each $j, 1 \le j \le n$
   b) $W_j \cap \sum_{i=1, i\neq j}^{n} W_i = \{0\}$ for atleast one $j, 1 \le j \le n$
   c) $W_j \cap \sum_{i=1, i\neq j}^{n} W_i = \{0\}$ for each $j, 1 \le j \le n$
   d) $\sum_{i=1, i\neq j}^{n} W_i = \{0\}$ for atleast one $j, 1 \le j \le n$

6) If λ is a characteristic value of a linear operator T, then the _________ multiplicity of λ is defined to be the multiplicity of λ as a root of the characteristic polynomial of T.
   a) Minimal polynomial b) Geometric
   c) Algebraic d) Unique

7) If $\dim V(F) = n$ then $\dim V^*(F) =$ _________, where V^* is the dual of V.
   a) n b) n^2
   c) 0 d) $n-1$

8) A form f on a complex vector space V is called Hermitian if $f(\alpha,\beta) =$ ________, for all $\alpha,\beta \in V$.
   a) $\overline{f(\alpha,\beta)}$ b) $f(\beta,\alpha)$
   c) $f(\alpha,\beta)$ d) $\overline{f(\beta,\alpha)}$

9) If T_1 and T_2 are the normal operators on an inner product space with property that either commutes with the adjoint of the other, then _______.
   a) T_1T_2 is not normal b) $T_1 + T_2$ is not normal
   c) $T_1 + T_2$ and T_1T_2 are not normal d) $T_1 + T_2$ and T_1T_2 are normal

10) Consider the statements:
   i) Product of two self adjoint operators on an inner product space is self adjoint.
   ii) Product of two unitary operators is unitary.
   a) Only I is true b) Only II is true
   c) Both I and II are true d) Both I and II are false

11) If A is an $n \times n$ matrix with characteristic polynomial $f(x) = (x-2)^2(x-3)^2(x-4)^2$, then trace (A)= ________.
   a) 9 b) 6
   c) 0 d) 29

12) If A is an n–square nilpotent matrix of index k, then its minimal polynomial is _________.
   a) x^{k-1} b) x^k
   c) x^{k+1} d) 0

13) If W_1 and W_2 be two subspaces of finite dimensional vector space $V(F)$ then $A(W_1 + W_2)$ is __________.
   a) $A(W_1) \cap A(W_2)$ b) $A(W_1) + A(W_2)$
   c) $A(W_1) - A(W_2)$ d) $A(W_1) \cup A(W_2)$

14) If W be a subspace of a vector space V, and T be a linear operator on V then W is said to be invariant under T if _________.
   a) $T(W) \subseteq W$ b) $T(W) \supseteq W$
   c) $T(W) = 0$ d) $T(W) = V$

Q.2 A) Answer the following questions. (Any Four) 08

1) Let T be a linear operator on an inner product space V. Then show that T is normal if $\|T(v)\| = \|T^*(v)\|$ for all $v \in V$.
2) If the characteristic polynomial of a 5×5 matrix B is $(x-2)^3(x+7)^2$, then find any two possible Jordan canonical forms of B.
3) If $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ then find the characteristic polynomial of A.
4) State primary decomposition theorem.
5) Let V be a finite dimensional inner product space and T and U are linear operators on V then show that $(TU)^* = U^*T^*$.

B) Write Notes. (Any Two) **06**

1) Orthogonal complement
2) Invariant subspace
3) Unitary operator

Q.3 A) Answer the following questions. (Any Two) **08**

1) If V is a finite dimensional vector space and $v \neq 0 \in V$, then show that there exists a functional $f \in V^*$ such that $f(v) \neq 0$.
2) If T is a linear operator on a finite dimensional inner product space V, then prove that there exists a unique linear operator T^* on V such that $(T(\alpha), \beta) = \big(\alpha, T^*(\beta)\big)$, for α, β in V.
3) Find the dual basis of the basis set $\mathbb{B} = \{(1,-1,3),(0,1,-1),(0,3,-2)\}$ for $V_3(\mathbb{R})$.

B) Answer the following question. (Any One) **06**

1) State and prove Cayley-Hamilton theorem.
2) If T is a linear operator on a finite dimensional inner product space V and W is a T –invariant subspace of V, then show that $W^\perp$ is invariant under T^*.

Q.4 A) Answer the following questions. (Any Two) **10**

1) If $\dim V(F)$ is finite and W is a subspace of V, then show that
$$\dim W + \dim A(W) = \dim V$$
2) If A is a $n \times n$ matrix with entries in the field F, and $P_1, P_2, \dots, P_r$ are the invariant factors for A, then show that the matrix $xI - A$ is equivalent to the $n \times n$ diagonal matrix with diagonal entries $P_1, P_2, \dots, P_r$.
3) Suppose $T : V \to V$ is a linear operator and $f(t) = g(t)h(t)$, where $f(t), g(t)$ and $h(t)$ are polynomials such that $f(T) = \hat{0}$ and $g(t), h(t)$ are relatively prime, then show that V is the direct sum of T-invariant subspaces U and W where $U = \ker g(T)$ and $W = \ker h(T)$.

B) Answer the following questions. (Any One) **04**

1) If T is a linear operator on an n- dimensional vector space V over F, then show that the characteristic and minimal polynomials for T have the same roots, except for multiplication.
2) If $\dim V(F)$ is finite and T is a linear operator on V. Then show that T is diagonalizable if the minimal polynomial for T has the form $P = (x - c_1)(x - c_2) \dots (x - c_k)$, where $c_1, c_2, \dots, c_k$ are the distinct element of F.

Q.5 Answer the following questions. (Any Two) **14**

a) Find the minimal polynomial and the rational from of the following 3×3 real matrix.
$$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

b) If $\dim V(F) = n$ and $\mathbb{B} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a basis for V. If $\{x_1, x_2, \dots x_n\}$ is any set of n scalars, then show that there exists a unique linear functional f on V such that $f(\alpha_i) = x_i, \forall\, i, i = 1, 2, \dots, n$.

c) Apply the Gram-Schmidt process to the vectors $\beta_1 = (3,0,4), \beta_2 = (-1,0,7)$ and $\beta_3 = (2,9,11)$ to obtain a orthonormal basis for $\mathbb{R}^3$, with the standard inner product.

SLR-JP-323

Seat No.	

Set **P**

M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Mathematics
DIFFERENTIAL GEOMETRY

Day & Date: Saturday, 09-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If V and W are vector field then ∇_v W = ________.
   a) $\sum_i V\,[Wi]\,Ui(p)$ b) $\sum_i W\,[Vi]$
   c) $\sum_i W\,[Vi]\;Ui(p)$ d) $\sum_i V\,[Wi]$

2) Shape operator of plane is ________.
   a) always zero b) 1
   c) -1 d) -1 to 1

3) Quadratic approximation of curve represents ________.
   a) circle b) straight line
   c) parabola d) None of these

4) If α is curve in E^3 then length function $s(t)$ of curve α from 0 to any point t on curve is given by $s(t) =$ ________.
   a) $\int_0^t \|\alpha'(u)\|\,du$ b) $\int_0^1 \|\alpha'(u)\|\,du$
   c) $\int_0^t \|\alpha(u)\|\,du$ d) $\int_0^t \|\alpha''(u)\|\,du$

5) If F is isometry of E^3 and $F(0) = 0$ then F is ________.
   a) Linear transformation b) Translation
   c) Orthogonal transformation d) None of these

6) If u, v, w are linearly dependent then ________.
   a) $u.(v \times w) \neq 0$ b) $u.(v \times w) = 0$
   c) $u \times (v \times w) \neq 0$ d) $u \times v \neq 0$

7) Mean curvature is given by $H(p) =$ ________.
   a) $\frac{trace(S)}{2}$ b) $trace\ (S)$
   c) $2\ trace\ (S)$ d) $(trace(S))^{1/2}$

8) If U_1, U_2, U_3 are natural frame field at p then $Ui\ [f] =$ ________.
   a) $\frac{df}{dx}$ b) $\frac{df}{dx_i}$
   c) $\frac{\partial f}{\partial x_i}$ d) $\frac{\partial f}{\partial x}$

9) Curve α is cylindrical helix if _________.
   a) $\tau = 0$ b) $k = 0$
   c) k =constant d) $\frac{\tau}{k}$ =constant

10) If $\sum$ is sphere of radius a with center at origin of E^3, then for any spherical curve β on sphere, curvature of β is _________.
   a) atmost $\frac{1}{a}$ b) at least $\frac{1}{a}$
   c) exactly $\frac{1}{a}$ d) exactly a

11) If T is translation by vector a then T^{-1} is translation by ________.
   a) 0 b) -a
   c) a d) Zero

12) A Frenet formulae for unit speed curve N' is equal to _________.
   a) $\tau B - kT$ b) $-\tau N$
   c) $\tau B + kT$ d) kT'

13) If $k > 0,\ \tau \neq 0$ for curve $\propto$ then curve α is _________.
   a) circular helix b) cylindrical helix
   c) circle d) sphere

14) Mapping $F : E^3 \rightarrow E^3$ defined by $F(P) = -P + 1$ is ________.
   a) Translation and isometry b) Not translation but isometry
   c) Neither translation nor isometry d) Translation but not isometry

Q.2 A) Answer the following questions. (Any Four) **08**

1) Define
   i) Tangent vector in E^3
   ii) Natural coordinates
2) i) Define Tangent space
   ii) Draw the vectors $\{(1,0,0)_p, (0,1,0)_p, (0,0,1)_p\}$
3) Compute velocity vector of curve
   i) $\alpha(t) = (t, 1 + t^2, t)\ \ at\ t = 0$
   ii) $\alpha(t) = (\sin t, \cos t, t)\ \ at\ t = 0$
4) Check $M : (x^2 + y^2) + 3z^2 = 1$ is surface or not.
5) Define
   i) Covariant derivative of vector field w.r.t. tangent vector
   ii) Covariant derivative of vector field w.r.t. vector field

B) Write Notes. (Any Two) **06**

1) i) Tangent vector for curve α
   ii) Normal vector
   iii) Binormal vector
2) Write a note on Reparametrization of a curve $\propto (t)$
3) Proper patch

Q.3 A) Answer the following questions. (Any Two) **08**

1) Find Gaussian curvature for $X(u, v) = (ucosv, usinv, bv)\quad b \neq 0$
2) If $W = xy^3U_1 - x^2y^2U_3$ and $V = (-1, 2, -1)\quad P \equiv (1, 3, 2)$ find
   i) $\nabla_v W$ ii) $\nabla_v(\nabla_v W)$
3) i) If S and T are Translation then show that $ST = TS$ is also translation.
   ii) If T is translation by a show that T^{-1} is translation by $-a$

B) Answer the following questions. (Any One) **06**

1) Define shape operator. Show that for each point p of $M \subseteq R^3$ shape operator is linear operator.
2) If $X : D \to E^3$ $X(u,v) = (u^2, v^2, uv)$ verify whether X is proper patch. where $D : \{(u,v) \in E^2 / u, v > 0\}$

Q.4 A) Answer the following questions. (Any Two) **10**

1) $\phi = xdx - ydy$ $\quad \psi = zdx + xdz$ $\quad \theta = zdy$
   Find
   i) $\phi \wedge \psi$ ii) $\psi \wedge \theta$ iii) $\phi \wedge \theta$ iv) $\phi \wedge \psi \wedge \theta$
2) $V = y^2 z\, U_1 + xy\, U_2 - 3xzU_3$ $\quad f(x,y,z) = xy$
   $g(x,y,z) = x^2 - 2z^2$ find
   i) $V[f]$ ii) $V[g]$ iii) $V[2f - 3g]$ iv) $V[fg]$
3) Show that shape operator of cylindrical surface is half flat & half round.

B) Answer the following questions. (Any One) **04**

1) Compute Tangent T, Normal N and curvature K for $\alpha(t) = (\cos ht, \sin ht, t)$.
2) Show that Rotation is an orthogonal transformation.

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that every isometry of E^3 can be uniquely described as orthogonal transformation followed by translation.

b) Find parametrization for surface obtained by revolving the profile curve $C : (z-2)^2 + y^2 = 1$ around Y-axis.

c) Obtain expression for frenet formulae for unit speed curve.

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
MEASURE AND INTEGRATION

Day & Date: Monday, 04-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Multiple Choice Questions. **14**

1) For a signed measure v defined on a measurable space $(X, \mathbb{B})$. Then for $E \in \mathbb{B}$, total variation $|v|(E) =$ ________.
   a) $v^+(E) + v^-(E)$
   b) $v^+(E) - v^-(E)$
   c) 0
   d) positive

2) Consider the two statements.
   i) If v is a measure $\Rightarrow v$ is a signed measure.
   ii) If v is a signed measure $\Rightarrow v$ is a measure. Then ________.
   a) Only i is true
   b) Only ii is true
   c) Both i and ii are true
   d) Both i and ii are false

3) A measure μ on a measurable space is a saturated if ________.
   a) Every subset of X is a measurable
   b) Every locally measurable subset of X is a measurable
   c) Every measurable subset of X is a locally measurable
   d) None of these

4) The set E is said to be measurable w.r.t. μ^* if for every set A ________.
   a) $\mu^*(E) = \mu^*(A \cap E) + \mu^*(A \cap E')$
   b) $\mu^*(E) = \mu^*(E \cap A) + \mu^*(E \cap A')$
   c) $\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap E')$
   d) $\mu^*(A) = \mu^*(E \cap A) + \mu^*(E \cap A')$

5) In Fubini's theorem, measures μ and v are ________.
   a) Finites
   b) Completes
   c) σ-finites
   d) semi-finites

6) A measurable set A is called null set w.r.t. a signed measure v if ________.
   a) $v(A) = 0$
   b) $v(A) \geq 0$
   c) every measurable subset E of A we have $v(E) = 0$
   d) every subset E of A we have $v(E) = 0$

7) If f is an integrable function defined on a measurable set E such that $f = 0$ almost everywhere then ________.
   a) $\int_E f d\mu \neq 0$
   b) $\int_E f d\mu = 0$
   c) $\int_E f d\mu \geq 0$
   d) $\int_E f d\mu \leq 0$

8) Consider the two statements.
   i) Lebesgue measure of $[0, 1]$ is infinite.
   ii) $[0, 1]$ is an uncountable subset of $\mathbb{R}$. Then ________.
   a) Only i is true
   b) Only ii is true
   c) Both i and ii are true
   d) Both i and ii are false

9) A measure space $(X, \mathbb{B}, \mu)$ is called complete, if $\mathbb{B}$ contains all subsets of a set of measure _________.
   a) Finite b) Infinite
   c) Zero d) One

10) The collection of measurable rectangles is ________.
   a) Algebra b) Semi-algebra
   c) σ-algebra d) None of these

11) If f_n is a sequence of non-negative measurable functions that converge almost every-where on a set E to a function f and suppose that $f_n \leq f$ for all n. Then _________.
   a) $\int_E f \leq \overline{lim} \int_E f_n$ b) $\int_E f \geq \overline{lim} \int_E f_n$
   c) $\int_E f \geq \underline{lim} \int_E f_n$ d) $\int_E f \leq \underline{lim} \int_E f_n$

12) Suppose $\mu^*(E) < \infty$. Consider the two statements.
   i) If E is measurable then $\mu_*(E) = \mu^*(E)$.
   ii) If $\mu_*(E) = \mu^*(E)$ then E is measurable. Then _______
   a) Only i is true b) Only ii is true
   c) Both i and ii are true d) Both i and ii are false

13) An outer measure μ^* is said to be regular if given any subset E of X and any $\epsilon \geq 0$, there is a μ^* −measurable set A with $E \subseteq A$ and ________.
   a) $\mu^*(A) \leq \mu^*(E) + \epsilon$ b) $\mu^*(A) \geq \mu^*(E) + \epsilon$
   c) $\mu^*(E) \leq \mu^*(A) + \epsilon$ d) $\mu^*(E) \geq \mu^*(A) + \epsilon$

14) Hahn decomposition is unique expect for _________.
   a) positive sets b) negative sets
   c) null sets d) measurable sets

Q.2 A) Answer the following questions. (Any Four) **08**

1) Show that the collection of locally measurable sets is a σ-algebra.
2) If f and g are measurable functions on a measure space $(X, \mathbb{B}, \mu)$ then prove that f^2 is measurable function.
3) If $E \subseteq F$ then show that $\mu_*(E) \leq \mu_*(F)$.
4) State Fubini' theorem.
5) Show that countable union of positive set is positive.

B) Write notes(Any Two) **06**

1) Signed measures
2) Product measure
3) Inner measure

Q.3 A) Answer the following questions .(Any Two) **08**

1) If $v_1 \perp \mu, v_2 \perp \mu$ then show that $c_1 v_1 + c_2 v_2 \perp \mu$ where v_1, v_2, μ are measures on measurable space $(X, \mathbb{B})$ and c_1, c_2 are constant.
2) If $(X, \mathbb{B}, \mu)$ is a measurable space and $A, B \in \mathbb{B}$ with $A \subseteq B$, then show that $\mu(A) \leq \mu(B)$.
3) If $\{A_n\}$ is a countable collection of measurable sets, then show that

$$\mu\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{n \to \infty} \mu\left(\bigcup_{k=1}^{n} A_k\right)$$

B) Answer the following questions. (Any One) **06**

1) If $(X, \mathbb{B}, \mu)$ is a σ-finite measure space and v is a σ-finite measure on $\mathbb{B}$, then show that we can find a measure v_0 mutually singular with respect to μ and measure v_1 absolutely continuous with respect to μ, such that $v = v_0 + v_1$. Also show that the measures v_0 and v_1 are unique.

2) State and prove Jordan decomposition theorem.

Q.4 A) Answer the following questions. (Any Two) **10**

1) Define a semi algebra. Prove that collection of all measurable rectangles $\mathbb{R}$ is a semi algebra.
2) Prove that if $A \in \mathbb{A}$ then
$$\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap E^c)$$
for any $E \subseteq X$ where μ_* is an inner measure and μ^* is an outer measure.
3) If v is a signed measure on the measurable space $(X, \mathbb{B})$ then show that there is a positive set A and negative set B such that $X = A \cup B$ and $A \cap B = \emptyset$

B) Answer the following questions. (Any One) **04**

1) State and prove Monotone convergence theorem.
2) If f, g are non-negative measurable functions and a, b are non-negative real numbers then prove that
$$\int (af + bg)d\mu = a\int f d\mu + b\int g d\mu$$

Q.5 Answer the following questions. (Any Two) **14**

a) State and prove Radon-Nikodym theorem for a σ-finite measure space by assuming it is true for finite measure space.

b) State and prove Tonelli theorem.

c) Define integrable function; and if f, g are integrable functions and E is a measurable set, then show that

1) $$\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g$$
2) If $|h| \leq |f|$ and h is measurable then h is integrable.
3) If $f \geq g$ a.e., then $\int f \geq \int g$

SLR-JP-326

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
PARTIAL DIFFERENTIAL EQUATION

Day & Date: Wednesday, 06-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) $a_0 \frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1}\partial y} + \cdots + a_n \frac{\partial^n z}{\partial y^n} = F(x, y)$ is ________.
   a) homogeneous pde
   b) non-homogeneous pde with constant coefficient
   c) non-homogeneous pde
   d) homogeneous pde with constant coefficient

2) The general solution of $Pp + Qq = R$ is __________.
   a) $\phi(u, v) = 1$ b) $\phi(u, v) = -1$
   c) $\phi(u, v) = 0$ d) $\phi(u, v) = c$

3) Second order partial differential equations are classified in to ________.
   a) hyperbolic type b) parabolic type
   c) elliptic type d) All of these

4) Order of the equation $ptany + qtany = sec^2z$.
   a) 2 b) 1
   c) 0 d) None of these

5) Eliminating a, b from $z = (x + a)(y + b)$ gives _________.
   a) $pq = z$ b) $\frac{p}{q} = z$
   c) $p + q = z$ d) None of these

6) The complete integral of the pde $z = px + qy + logpq$
   a) $z = x + y$ b) $z = ax + by + logab$
   c) $z = ax + by$ d) None of these

7) Integral of $yzdx + xzdy + xydz = 0$ is _________.
   a) $xyz = 0$ b) $xyz = c$
   c) $x + y + z = c$ d) None of these

8) Complete integral of $z^2(1 + p^2 + q^2) = 1$ is $(x - a)^2 + (y - b)^2 + z^2 = 1$
   a) true b) false
   c) a) and b) d) None of these

9) A two parameter family of solutions $z = F\,(x, y, a, b)$ is called complete integral if the rank of the matrix $\begin{bmatrix} F_a & F_{xa} & F_{ya} \\ F_b & F_{xb} & F_{yb} \end{bmatrix}$ is _________.
   a) two b) one
   c) three d) none of these

10) Suppose that $u(x, y)$ is harmonic in a bounded domain D and is continuous on $\bar{D} = D \cup B$, where B is boundary of D. Then $u(x, y)$ attains its minimum _________.
   a) on B b) inside D but not on B
   c) outside D but not on B d) inside D as well as on B

11) The solution of Neumann problem differs by ________.
   a) function of x b) function of y
   c) function of x and y d) constant

12) If there is a functional relation between two functions $u(x, y)$ and $v(x, y)$ not involving x and y explicitly then ________.
   a) $\frac{\partial u}{\partial x} = 0 \; and \frac{\partial v}{\partial y} \neq 0$ b) $\frac{\partial v}{\partial x} = 0 \; and \frac{\partial u}{\partial y} \neq 0$
   c) $\frac{\partial(u, v)}{\partial(x, y)} = 0$ d) $\frac{\partial(u, v)}{\partial(x, y)} \neq 0$

13) The first order pde which is linear in p, q & z is known as _________.
   a) Linear b) Semi linear
   c) Quasi linear d) None of these

14) A $pde (n-1)^2 u_{xx} - y^{2n} u_{yy} = n y^{2n-1} u_y$ is of hyperbolic type if (Where n is an integer)
   a) $n = 1$ b) $n < 1$
   c) $n > 1$ d) $n = 0$

Q.2 A) Answer the following questions. (Any Four) 08
1) From the PDE by eliminating arbitrary function from $F(xyz,\; x + y + z) = 0$
2) Write auxiliary equation of Charpits method.
3) When we say that Pfaffian differential equation is exact?
4) What is complete integral of $pq = 1$?
5) Define compatible system of first order PDE.

B) Write Notes on. (Any Two) 06
1) Define complete integral and general integral.
2) Write a note on characteristic strip.
3) Discuss the case of second order parabolic type equation.

Q.3 A) Answer the following questions. (Any two) 08
1) Solve $u_{xx} - u_{tt} = 0$.
2) Prove that there always exists an integrating factor for Pfaffian differential equation in two variables.
3) Find the integral of $ydx + xdy + 2zdz = 0$

B) Answer the following questions. (Any One) 06
1) Let $u(x, y)$ and $v(x, y)$ be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u,v)}{\partial(x,y)} = 0$, then prove that there exist a relation between u and v not involving x and y explicity.
2) Show that the surfaces $f(x, y, z) = x^2 + y^2 + z^2 = c, c > 0$ can form an equipotential family of surfaces.

Q.4 A) Answer the following questions. (Any Two) 10
1) Explain analytic expression for the Monge cone at (x_0, y_0, z_0)
2) Prove that general solution of Lagranges equation is $F(u, v) = 0$ where F is an arbitrary differential function of $u(x, y, z)$ and $v(x, y, z)$.
3) Solve $xu_x + yu_y = u_z^2$ by using Jacobi's method.

B) Answer the following questions. (Any One) **04**

1) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x), x \geq 0$
2) Show that the solution of the Dirichlet problem if it exists is unique.

Q.5 Answer the following questions. (Any two) **14**

a) Prove that a necessary and sufficient condition that the pfaffian differential equation $\bar{X} \cdot d\bar{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ be integrable is that $\bar{X} \cdot curl\bar{X} = 0$.

b) State and prove Harnack's theorem.

c) Reduce the equation $(n-1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} u_y$ where n is an integer, to a canonical form.

SLR-JP-327

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
INTEGRAL EQUATIONS

Day & Date: Friday, 08-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Multiple Choice Questions. **14**

1) Which of the following is convolution property?
a) $f * g = \int_0^t f(u)\, g(t-u)du$
b) $f * g = \int_0^t f(u)\, g(t+u)du$
c) $f * g = \int_0^t f(t-u)\, g(t-u)du$
d) $f * g = \int_0^t f(u)\, g(t)du$

2) If $L[f(t)] = F(s)$ then $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}\,(u)dv$ provided that _________.
a) $\lim_{t\to\infty} \frac{f(t)}{t}\ exist$
b) $\lim_{t\to\infty} \frac{f(t)}{t}\ does\ not\ exist$
c) $\lim_{t\to\infty} \frac{f(t)}{t} = 1$
d) $\lim_{t\to\infty} \frac{f(t)}{t} = 0$

3) Set of eigenvalues of second iterated kernel coincide with set of squares of eigenvalues of __________ kernel.
a) Given
b) Symmetric
c) Separable
d) Iterated

4) The eigenfunctions of symmetric kernel, corresponding to different eigen values are __________.
a) Real
b) Imaginary
c) Orthogonal
d) Equal

5) The integral equation $e^t + 2\int_0^1 e^{(t-s)^2} x(s) = 0$ is a _________.
a) linear fredholm integral equation of second kind
b) linear volterra integral equation of second kind
c) linear fredholm integral equation of first kind
d) linear volterra integral equation of first kind

6) Value of integral $\int_0^t J_0(t)\, J_0(t-s)ds$ J_0 - is Bessel function of zero order is _______.
a) e^t
b) e^{-t}
c) $\sin t$
d) $\cos t$

7) The Resolvent kernel is given by ________.

a) $\sum_{m=1}^{\infty} \lambda^{m-1}\, k_m(x,t)$
b) $\sum_{m=1}^{\infty} \lambda^{m}\, k_m(x,t)$
c) $\sum_{m=0}^{\infty} \lambda^{m-1}\, k_m(x,t)$
d) $\sum_{m=0}^{\infty} \lambda^{m}\, k_m(x,t)$

8) $(x^2t^2 + xt + 1)$ is ________ kernel.

a) Symmetric
b) Separable
c) Convolution
d) None of these

9) A kernel $k(x,t)$ is called degenerated if _________.

a) $\sum_{i=1}^{n} gi(x)\, hi(t)$
b) $\sum_{i=1}^{n} gi(x)\, hi(x)$
c) $\sum_{i.j=1}^{n} gi(x)\, hj(t)$
d) $\sum_{i=1}^{n} [gi(x)]^2$

10) Which of the following is not symmetric kernel?

a) $\sin(x+t)$
b) $\mathrm{i}(x+t)$
c) $x^2t^3 + 1$
d) $\log(xt)$

11) By solving integral value problem we obtain _________.

a) Volterra integral equation
b) Linear Volterra integral equation
c) Non Linear Volterra integral equation
d) fredholm integral equation

12) If $k(x,t) = x - 2t$ $\quad x, t \in [0, 2\pi]$ $\quad k_2(x,t)$ _________.

a) $\left[\frac{x}{2} - 2tx + 2t - \frac{2}{3}\right]$
b) $\left[\frac{x}{2} - 2tx + 2\right]$
c) $\left[\frac{x}{2} - 2tx + 2t\right]$
d) $\left[\frac{x}{2} - 2tx^2\right]$

13) $g(x)y(x) = f(x) + \lambda \int_a^b k(x,t)\, y(t)dt$ is homogenous fredholm integral equation of second kind if _______.

a) $g(x) = 1 \quad f(x) \neq 0$
b) $g(x) = 1 \quad f(x) = 0$
c) $g(x) \neq 1 \quad f(x) = 0$
d) $g(x) = 1 \quad f(x) = 1$

14) For fredholm integral equation of a and b are limit of integration then a = _______ and b = _________.

a) a, b = constant
b) a = constant b = variable
c) a = variable b = constant
d) a, b = variable

Q.2 A) Answer the following (Any Four) **08**

1) State Leibnitz Rule
2) i) Define convolution kernel
   ii) Formula for converting multiple integral into single integral
3) Solve integral equation $y(x) = \frac{1}{2}\int_0^{\pi} \sin x\, y(t)dt$
4) Solve $t = \int_0^t e^{t-s}\, y(s)ds$ using laplace transform

5) Solve $\sin t = \int_0^t J_0(t-x)Y(x)dx$

B) Answer the following (Any Two) **06**

1) Show that $Y(x) = \frac{2e^x}{3}(x-1)$ is solution of

$$y(x) + 2\int_0^1 e^{x-t}\, y(t)dt = [x-2]\frac{2e^x}{3}$$

2) Find Resolvent kernel of $k(x,t) = xt$
3) Find $k_2(x,t), k_3(x,t)$ for $k(x,t) = (x-t)^2 \quad a=-1 \quad b=1$

Q.3 A) Answer the following (Any Two) **08**

1) Solve

$$Y(t) = a\sin t - 2\int_0^1 Y(x)\cos(t-x)dx$$

2) Find Greens function of $y'' = 0 \quad y(0) = y(l) = 0$
3) Write types of volterra integral equation.

B) Answer the following (Any One) **06**

1) Convert Initial value problem
$y''' + xy'' + (x^2-x)y = xe^x + 1 \qquad y(0) = 1 = y'(0), \quad y''(0) = 0$
to integral equation
2) Find eigen values and eigen functions of homogenous integral equation

$$y(x) = \lambda\int_{-1}^{1}(5xt^3 + 4x^2t + 3xt)y(t)dt$$

Q.4 A) Answer the following (Any Two) **10**

1) Solve $xy'' + y' = 0 \qquad y(0) = 1 \qquad y(l) = 0$
2) Find solution of $y(x) = e^x + \lambda\int_0^{10} x + y(t)dt$ using resolvent kernel
3) If a kernel is symmetric then prove that all its iterated kernel are also symmetric.

B) Answer the following (Any One) **04**

1) Prove that eigen values of symmetric kernel are real.
2) Find n^{th} iterated kernel of $k(x,t) = e^{-(x-t)}$

Q.5 Answer the following (Any Two) **14**

a) Find iterated kernel of $k(x,t) = (x + \sin t)\ a = -\pi \ and\ b = \pi$
b) Explain solution of volterra integral equation of second kind by successive approximations
c) Solve $y'' + y = x \quad y(0) = 0, \quad y'(1) = 0$ using Greens function.

SLR-JP-328

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
OPERATIONS RESEARCH

Day & Date: Monday, 11-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) A simplex in two dimension is a ______.
   a) triangle b) line
   c) Point d) plane

2) The convex null of X is the ______ convex set containing X.
   a) Largest b) Maximal
   c) Smallest d) None

3) The intersection of finite number of closed half spaces in R^n is called _______.
   a) convex null b) polyhedral convex set
   c) convex cone d) simplex

4) The set of variables is said to be feasible solution if ________.
   a) It satisfies constraints only
   b) It satisfies constraints and non-negative restrictions
   c) It satisfies non-negative restrictions only
   d) None of these

5) Simplex method is iterative method to solve ______ programming problem.
   a) non-linear b) quadratic
   c) Linear d) none of these

6) The non-negative variable which is added to the left hand side of constraint to convert it into strict equation is called _______ variable.
   a) Surplus b) Slack
   c) Artificial d) None of these

7) For maximum problem, the coefficient of artificial variable in the objective function is _______.
   a) $+M$ b) $-M$
   c) $+1$ d) > 0

8) Beals method is used to solve ______ programming problem.
   a) non-linear b) linear
   c) Quadratic d) none of these

9) Branch and bound method is used to solve _______.
   a) Linear programming problem
   b) Convex programming problem
   c) Non-linear programming problem
   d) Integer programming problem

10) If values of all variables of an IPP are either 0 or 1 then problem is called _________.
    a) Mixed IPP b) All IPP
    c) IPP d) None of these

11) If either the primal or dual problem has an unbounded solution then the other has ______ solution.
    a) Feasible b) infeasible
    c) Unbounded d) optimal

12) In dual simplex method, _______ variables are not required.
    a) Slack b) surplus
    c) Original d) artificial

13) QPP is concerned with the NLPP of optimizing quadratic objective function subject to _______.
    a) linear inequality constraints b) non-Linear inequality constraints
    c) non-Linear equality constraints d) no constraints

14) A two person game is said to be zero sum if _______.
    a) Gain of one player is exactly matches by a loss to the other player so that their sum is zero.
    b) Both players must have exact number of strategies
    c) Diagonal entries of pay off matrix are zero
    d) Gain of the one layer does not match to the loss of other player

Q.2 A) Attempt any four of the following questions. **08**
1) If S and T are two convex sets in R^n then prove that S-T is convex set.
2) Explain the difference between simplex and dual simplex method.
3) Write matrix form of standard primal and standard dual LPP.
4) Write general form of QPP.
5) Define
   i) Two persons zero sum game.
   ii) Saddle point.

B) Attempt any two of the following questions. **06**
1) Define
   i) Slack variable.
   ii) Surplus variable.
   iii) Artificial variable.
2) If $\hat{X}$ is feasible solution to the primal and $\widehat{W}$ is feasible solution to the dual such that $C\hat{X} = b^T\widehat{W}$ then prove that $\hat{X}$ is the optimal solution to the primal and $\widehat{W}$ is an optimal solution to the dual.
3) Write the general rules for converting standard primal into its dual.

Q.3 A) Attempt any two of the following questions. **08**
1) Let S and T be two convex sets in R^n then prove that $\alpha S + \beta T$ is convex set for $\alpha, \beta \in R$.
2) Write algorithm of Beals method.
3) Explain pure and mixed strategy.

B) Attempt any one of the following questions. **06**
1) Solve the following LPP by using Big-M method.

   Max $z = -2x_1 - x_2$
   Subject to
   $3x_1 + x_2 = 3$
   $4x_1 + 3x_2 \geq 6$
   $x_1 + 2x_2 \leq 4$
   and $x_1, x_2 \geq 0$

2) Write algorithm of Gomory's cutting plane.

Q.4 A) Attempt any two of the following questions. **10**

1) Solve the following LPP

Max $z = 3x_1 + 5x_2$
Subject to
$x_1 + 2x_2 \leq 2000$
$x_1 + x_2 \leq 1500$
$x_2 \leq 600$
and $x_1, x_2 \geq 0$

2) If p^{th} variable of the primal is unrestricted in sign then prove that p^{th} constraint of the dual is an equation.

3) Explain How to construct Kuhn-tucker condition for QPP.

B) Attempt any one of the following questions. **04**

1) Explain graphical method to solve LPP with two variables.
2) Write short note on QPP.

Q.5 Attempt any two of the following questions. **14**

a) Solve the following LPP.

Max $z = 7x_1 + 9x_2$
Subject to
$-x_1 + 3x_2 \leq 6$
$7x_1 + x_2 \leq 35$
and $x_1, x_2 \geq 0$ are integers.

b) Apply Wolf's method to solve the following QPP.

Max $z = 2x_1 + x_2 - x_1^2$
Subject to
$2x_1 + 3x_2 \leq 6$
$2x_1 + x_2 \leq 4$
and $x_1, x_2 \geq 0$

c) If X_0 is an optimal solution to the primal then prove that there exist a feasible solution W_0 to the dual such that $CX_0 = b^T W_0$

SLR-JP-329

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
NUMERICAL ANALYSIS

Day & Date: Thursday, 14-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of calculator is allowed.

Q.1 Fill in the blanks by choosing correct alternatives given below. 14

1) The effect of error ______ with order of the differences.
   a) constant b) decreases
   c) increases d) none

2) Convergence of bisection method is ______.
   a) quadratic b) cubic
   c) very slow d) none

3) First approximation to the root of the equation $x^3 - 2\mathrm{x} - 5 = 0$ using method of false position is _______.
   a) 2.05882 b) 2.5882
   c) 2.15882 d) 2.882

4) Newton's ______ difference interpolation formula is useful for interpolation near the end of the tabular values.
   a) forward b) backward
   c) central d) none

5) In false position method, we choose two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of _______.
   a) opposite signs b) same signs
   c) constant d) none

6) The backward difference operator is _______.
   a) $\nabla f(x_i) = f(x_i + h) - f(x_i)$ b) $\nabla f(x_i) = f(x_i) - f(x_i - h)$
   c) $\nabla f(x_i) = f(x_i - h) - f(x_i)$ d) $\nabla f(x_i) = f(x_i) + f(x_i - h)$

7) Lagrange's formula is applicable if _______.
   a) Values of argument x are not equally spaced
   b) Values of argument x are equally spaced
   c) Corresponding values of y are not equally spaced
   d) None of these

8) Householders method is used to obtain Eigen values of ______ matrices.
   a) upper triangular b) lower triangular
   c) symmetric d) none of these

9) Which of the following is correct?
   a) $\nabla - \Delta = \Delta\nabla$ b) $\nabla - \Delta = -\Delta\nabla$
   c) $\nabla + \Delta = -\Delta\nabla$ d) $\nabla + \Delta = \Delta\nabla$

10) Which applying Simpson's 1/3 rule the number of subintervals should be _____.
   a) multiples of 5 b) odd
   c) even d) none

11) If A is upper triangular then A^{-1} is _______.
   a) lower triangular   b) upper triangular
   c) constant   d) none

12) In Newton Raphson's method the iterative formula to find $\frac{1}{N}$ is given by ______.
   a) $X_{n+1} = X_n(2 - NX_n)$ b) $X_{n+1} = X_n(2 + NX_n)$
   c) $X_{n+1} = -X_n(2 - NX_n)$ d) none

13) If i) $E^{-1}\nabla = \nabla - \nabla^2$ and
   ii) $\Delta\nabla = \delta^2$ then _____.
   a) Both (i) and (ii) true   b) Both (i) and (ii) false
   c) i) false and (ii) true   d) (i) true and (ii) false

14) A polynomial $p(x)$ is such that $p(0) = 5, p(1) = 10, p(2) = 17, p(3) = 26,$ $p(4) = 37,$ and $p(5) = 50$. The degree of $p(x)$ is _______.
   a) 5   b) 6
   c) 2   d) None of these

Q.2 A) Answer the following questions. (Any Four) **08**

1) Define rate of convergence of an iterative method.
2) Define absolute and relative errors.
3) Write Newton's backward difference interpolation formula.
4) Prove that $E = 1 + \Delta$
5) An approximate value of $\prod$ is 3.1428571 and its true value is 3.1415926. Find the absolute and relative errors.

B) Answer the following questions. (Any Two) **06**

1) Find the cubic polynomial which takes the values $y(0) = 1, y(1) = 0,$ $y(2) = 1$ and $y(3) = 10$.
2) Evaluate the sum $s = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors.
3) What is the difference between Secant and false position method?

Q.3 A) Answer the following questions. (Any Two) **08**

1) Find a positive root between 0 and 1 of the equation $xe^x = 1$ using iteration method up to three iterations.
2) Show that by using separation of symbols.
$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$$
3) Construct the divided difference table for the following tabular values.

X	2	4	6	8	10
Y=f(x)	10	20	30	40	50

B) Answer the following questions. (Any One) **06**

1) Derive Newton's backward difference interpolation formula.
2) Using divided difference, find $f(x)$ as polynomial in x from the following data.

X	-1	0	3	6	7
F(x)	3	-6	39	822	1611

Q.4 A) Answer the following questions. (Any Two) **10**

1) Show that Newton Raphson method converges quadratically.
2) Find real root of the equation $x^3 + x - 1 = 0$ by using bisection method.
3) Use Secant method to determine the root of the equation $\cos x - xe^x = 0$

B) Answer the following questions. (Any One) **04**

1) Explain method of false position.
2) Solve $\int_0^1 \frac{1}{1+x} dx$ correct to three decimal places by Simpsons $\frac{1}{3}$ rule with h=0.125

Q.5 Answer the following questions. (Any Two) **14**

a) Derive Newton's general interpolation formula with divided differences.

b) Determine the value of y using Modified Euler method when $x = 0.1$ given that $y(0) = 1,\ h = 0.05$ and $y' = x^2 + y$

c) Solve the following by Gauss Seidal method.

$$10x + 2y + z = 9$$
$$x + 10y - z = -22$$
$$-2x + 3y + 10z = 22$$

SLR-JP-331

Seat No.	

Set P

M.Sc. (Semester - IV) (CBCS) Examination Oct/Nov-2019
Mathematics
PROBABILITY THEORY

Day & Date: Thursday, 14-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternative given below. **14**

1) The sequence of sets $\{A_n\}$, where $A_n = \left(2, 3, +\frac{1}{n}\right), n \in N$ converges to the sets ______.
   a) (2, 3) b) empty set
   c) [2, 3] d) (2, 3]

2) The probability measure is always _______.
   a) non-negative b) normed
   c) additive d) all of these

3) Which of the following mode of convergence implies convergence in distribution?
   a) convergence almost sure b) convergence in probability
   c) convergence in quadratic mean d) all of these

4) For a random variable symmetric around zero, the characteristic function is _______.
   a) real-valued b) even
   c) both a and b d) none of these

5) If P is a probability measure, and if for disjoint sets A and B, $P(A) = 0.2$, $P(B) = 0.4$ then $P(A \cap B) =$ _______.
   a) 0.6 b) 0.5
   c) 0.2 d) 0

6) If X is a random variable that takes value 2 with probability 1, then characteristic function of X equals _______.
   a) e^{it} b) itc
   c) $\mathrm{i(c + t)}$ d) $\mathrm{e}^{2\mathrm{it}}$

7) The Lebesgue measure is always _______.
   a) finite b) normed
   c) non-negative d) none of these

8) Which of the following is the weakest convergence?
   a) convergence in probability b) convergence almost sure
   c) convergence in distribution d) convergence in r^{th} mean

9) If A^C contains finite number of elements, then set A is called as ______.
   a) nearly finite b) slightly finite
   c) C-finite d) none of these

10) Expectation of a non-negative random variable is _______.
   a) always non-negative b) may or may not be non-negative
   c) always negative d) none of these

11) If the set A has measure (μ) zero, then is _______.
   a) empty set
   b) μ- null set
   c) both a and b
   d) none of the above

12) Which of the following is a simple random variable?
   a) Poisson r.v.
   b) Geometric r.v.
   c) Discrete uniform r.v.
   d) None of these

13) The necessary and sufficient condition for a random variable X to be integrable is _______.
   a) X has to be non-negative
   b) X should be symmetric
   c) |X| has to be integrable
   d) X should be simple

14) A finite linear combination of indicators of sets is called ______ function.
   a) simple
   b) elementary
   c) arbitrary
   d) none of these

Q.2 A) Answer the following questions. (Any Four) **08**

1) State monotone convergence theorem.
2) Define distribution function of a random variable.
3) Define a class closed under countable intersections. Illustrate with an example.
4) State Lindeberg-Feller theorem on CLT.
5) Define indicator function.

B) Write Notes. (Any Two) **06**

1) Give axiomatic definition of probability measure
   Also prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
2) Is union of two fields always a field? Justify your answer.
3) Discuss σ-field induced by r.v. X.

Q.3 A) Answer the following questions. (Any Two) **08**

1) Discuss the convergence of sequence of sets in terms of limsup and liminf.
2) Prove or disprove: Inverse mapping preserves all set relations.
3) Show that every σ-field is field, but every field may not be a σ-field.

B) Answer the following questions. (Any One) **06**

1) State and prove Yule-Slutsky results.
2) State and prove Fatou's lemma.

Q.4 A) Answer the following questions. (Any Two) **10**

1) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables.
2) Define :
   i) Convergence in probability
   ii) Convergence in distribution
   iii) Convergence in r^{th} mean
3) Define expectation of simple random variable. Also prove its linearity and scale preserving property.

B) Answer the following questions. (Any One) **04**

1) Define the characteristic function of a random variable. Also state its inversion theorem and uniqueness property.
2) Prove any three properties of characteristic function.

Q.5 Answer the following questions. (Any Two) **14**

a) Prove that inverse image of σ-field is also a σ-field.
b) Define limit inferior and limit superior for a sequence of random variables. When do we say that sequence converges?
c) Write a short note on independence of random variables. Illustrate with examples.