

**PUNYASHLOK AHILYADEVJI HOLKAR  
SOLAPUR UNIVERSITY, SOLAPUR**



**Name of the Faculty: Science & Technology**

**CHOICE BASED CREDIT SYSTEM**

**Syllabus: STATISTICS**

**Name of the Course: M.Sc. Part-I**

**(Syllabus to be implemented from w.e.f. June 2020)**

**PUNYASHLOK AHILYADEVI HOLKAR SOLAPUR UNIVERSITY**

**SCHOOL OF COMPUTATIONAL SCIENCES**

**DEPARTMENT OF STATISTICS**

**Syllabus of M.Sc. Statistics (Choice Based Credit System)**

- 1) **Title of the course:** M.Sc. in Statistics.
- 2) **Duration of the course:** Two years.
- 3) **Pattern:** Choice Based Credit System (CBCS)
- 4) **Eligibility:** For M. Sc. in Statistics following candidates are eligible.
  - (i) B.Sc. with Statistics at principal level.
  - (ii) B.Sc. with Mathematics at principal and Statistics at subsidiary level.
- 5) **Intake Capacity:** 30

M. Sc. program in Statistics consists of 100 credits. Credits of a course are specified against the title of the course.

**A Four Semester M.Sc. Statistics Course**

| <b>Semester</b>   | <b>No. of Papers/<br/>Practicals /<br/>Seminar</b> | <b>Marks</b>          | <b>Credits</b>       |
|---|--|-----------------------|----------------------|
| <b>Semester I</b> <ul style="list-style-type: none"><li>• Theory Papers</li><li>• Practical Papers</li><li>• Seminar/Tutorial/Home Assignment /Field Tour/ Industrial Visit</li></ul>                     | 05<br>02<br>01                                     | 500<br>100<br>25      | 20<br>04<br>01       |
| <b>Semester II</b> <ul style="list-style-type: none"><li>• Theory Papers</li><li>• Practical Papers</li><li>• Seminar/ Tutorial/Home Assignment /Field Tour/ Industrial Visit</li></ul>                   | 05<br>02<br>01                                     | 500<br>100<br>25      | 20<br>04<br>01       |
| <b>Semester III</b> <ul style="list-style-type: none"><li>• Theory papers</li><li>• Practical Papers</li><li>• Seminar/ Tutorial/Home Assignment /Field Tour/ Industrial Visit</li></ul>                  | 05<br>02<br>01                                     | 500<br>100<br>25      | 20<br>04<br>01       |
| <b>Semester IV</b> <ul style="list-style-type: none"><li>• Theory papers</li><li>• Practical Papers</li><li>• Project</li><li>• Seminar/ Tutorial/Home Assignment /Field Tour/ Industrial Visit</li></ul> | 05<br>01<br>01<br>01                               | 500<br>50<br>50<br>25 | 20<br>02<br>02<br>01 |
| <b>Total marks and credits for M.Sc. Course</b>   |  | <b>2500</b>           | <b>100</b>           |

**Punyashlok Ahilyadevi Holkar Solapur University, Solapur**  
**M.Sc. Part-I (Statistics) CBCS Structure w.e.f. 2020-21**

| <b>M.Sc. STATISTICS SEMESTER-I</b>       |  |                      |            |            |           |           |           |           |
|--|--|----------------------|------------|------------|-----------|-----------|-----------|-----------|
| Paper Code                               | Title of the Paper                     | Semester Examination |            |            | L         | T         | P         | Credits   |
|  |  | Theory               | IA         | Total      |           |           |           |           |
| <b>Hard Core Theory</b>                  |  |                      |            |            |           |           |           |           |
| HCT 1.1                                  | Real Analysis                          | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| HCT 1.2                                  | Linear Algebra and Linear Models       | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| HCT 1.3                                  | Distribution Theory                    | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| HCT 1.4                                  | Estimation Theory                      | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| <b>Soft Core-Theory (Any one)</b>        |  |                      |            |            |           |           |           |           |
| SCT 1.1                                  | Statistical Computing                  | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| SCT 1.2                                  | Demography                             |                      |            |            |           |           |           |           |
| <b>Practical</b>                         |  |                      |            |            |           |           |           |           |
| HCP1.1                                   | PRACTICAL-1<br>(Based on HCT 1.2 1.4)  | 40                   | 10         | 50         | --        | --        | 6         | 2         |
| HCP1.2                                   | PRACTICAL-2<br>(Based on HCT 1.3, SCT) | 40                   | 10         | 50         | --        | --        | 6         | 2         |
|  | Seminar / Tutorial                     | ---                  | 25         | 25         | --        | 1         | --        | 1         |
| <b>Total for Semester-I</b>              |  | <b>480</b>           | <b>145</b> | <b>625</b> | <b>--</b> | <b>--</b> | <b>--</b> | <b>25</b> |
| <b>M.Sc. STATISTICS SEMESTER-II</b>      |  |                      |            |            |           |           |           |           |
| Code                                     | Title of the Paper                     | Semester Examination |            |            | L         | T         | P         | Credits   |
|  |  | Theory               | IA         | Total      |           |           |           |           |
| <b>Hard Core Theory</b>                  |  |                      |            |            |           |           |           |           |
| HCT 2.1                                  | Probability Theory                     | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| HCT 2.2                                  | Stochastic Processes                   | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| HCT 2.3                                  | Theory of Testing of Hypotheses        | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| <b>Soft Core Theory (Any One)</b>        |  |                      |            |            |           |           |           |           |
| SCT 2.1                                  | Sampling Theory                        | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| SCT 2.2                                  | Actuarial Statistics                   |                      |            |            |           |           |           |           |
| <b>Open Elective Theory (Any one)</b>    |  |                      |            |            |           |           |           |           |
| OET 2.1                                  | Statistical Methods                    | 80                   | 20         | 100        | 4         | --        | --        | 4         |
| OET 2.2                                  | Mathematical Statistics                |                      |            |            |           |           |           |           |
| <b>Practical (Hard and Soft core)</b>    |  |                      |            |            |           |           |           |           |
| HCP 2.1                                  | PRACTICAL-3<br>(based on HCT and SCT)  | 40                   | 10         | 50         | --        | --        | 6         | 2         |
| <b>Practical (Open Elective) Any One</b> |  |                      |            |            |           |           |           |           |
| OEP 2.1                                  | PRACTICAL-4: (based on OEP 2.1)        | 40                   | 10         | 50         | --        | --        | 6         | 2         |
| OEP 2.2                                  | PRACTICAL-4: (based on OEP 2.2)        |                      |            |            |           |           |           |           |
|  | Seminar / Tutorial                     | ---                  | 25         | 25         | --        | 1         | --        | 1         |
| <b>Total for Semester-II</b>             |  | <b>480</b>           | <b>145</b> | <b>625</b> | <b>--</b> | <b>--</b> | <b>--</b> | <b>25</b> |

HCT: Hard Core Theory

HCP: Hard Core Practical

SCT: Soft Core Theory

OET: Open Elective Theory

OEP: Open Elective Practical

IA: Internal Assessment

L:Lecture

T:Tutorials

P:Practical

**Evaluation Scheme:**

Each theory paper will have 100 marks out of which 80 marks will be for Term End examination and 20 marks for Internal Assessment. The candidate has to appear for internal evaluation of 20 marks and external evaluation (University Examination) of 80 marks for each theory paper.

Each practical paper will have 50 marks out of which 40 marks will be for Term End examination and 10 marks for Internal Assessment. The candidate has to appear for internal evaluation of 10 marks and external evaluation (University Examination) of 40 marks for each practical paper.

**Internal Evaluation:**

- In case of theory papers internal examinations will be conducted by department / school.
- In case of practical papers, 5 marks shall be for day-to-day journal and 5 marks shall be for internal test, which will be conducted by the department / school.

**External Evaluation (End of Term University Examination):**

**I) Nature of Theory question paper:**

- 1) Each Theory paper is of 80 marks.
- 2) Each Theory paper will be of 3 hours duration.

**II) Nature of Practical question paper: (End of Term Examination)**

**Sem-I and II:** Practical examination will be conducted for 40 marks and is of two hours duration. There shall be 05 questions each of 10 marks, of which student has to attempt any 03 questions. VIVA will be for 10 marks.

**Equivalence in accordance with titles and contents of theory papers for old and new syllabus.**

| <b>M.Sc. Statistics Semester-I</b>  |                                 |                     |                                  |
|-------------------------------------|---------------------------------|---------------------|----------------------------------|
| <b>Old Syllabus</b>                 |                                 | <b>New Syllabus</b> |                                  |
| Paper Code                          | Title of Paper                  | Paper Code          | Title of Paper                   |
| HCT 1.1                             | Real Analysis                   | HCT 1.1             | Real Analysis                    |
| HCT 1.2                             | Linear Algebra                  | HCT 1.2             | Linear Algebra and Linear Models |
| HCT 1.3                             | Distribution Theory             | HCT 1.3             | Distribution Theory              |
| HCT 1.4                             | Estimation Theory               | HCT 1.4             | Estimation Theory                |
| SCT 1.1                             | Statistical Computing           | SCT 1.1             | Statistical Computing            |
| SCT 1.2                             | Demography                      | SCT 1.2             | Demography                       |
| <b>M.Sc. Statistics Semester-II</b> |                                 |                     |                                  |
| HCT 2.1                             | Probability Theory              | HCT 2.1             | Probability Theory               |
| HCT 2.2                             | Stochastic Processes            | HCT 2.2             | Stochastic Processes             |
| HCT 2.3                             | Theory of Testing of Hypotheses | HCT 2.3             | Theory of Testing of Hypotheses  |
| SCT 2.1                             | Sampling Theory                 | SCT 2.1             | Sampling Theory                  |
| SCT 2.2                             | Actuarial Statistics            | SCT 2.2             | Actuarial Statistics             |
| OET 2.1                             | Statistical Methods             | OET 2.1             | Statistical Methods              |
| OET 2.2                             | Mathematical Statistics         | OET 2.2             | Mathematical Statistics          |

**M.Sc. Part-I (STATISTICS) SEMESTER-I**

|                  |                               |                           |
|------------------|-------------------------------|---------------------------|
| <b>Hard Core</b> | <b>HCT 1.1: REAL ANALYSIS</b> | <b>No. of credits: 04</b> |
|------------------|-------------------------------|---------------------------|

**Unit-1:** Set of real numbers, countable and uncountable sets, countability of rationals and uncountability of the interval  $(0, 1)$ , Supremum and Infimum of bounded sets, limit point of a set, open, closed, dense and compact sets. Bolzano- Weierstrass and Heine-Borel Theorems (Statements only). Applications of these theorems. (15L)

**Unit-2:** Sequence of real numbers, convergence, divergence, Cauchy sequence. Convergence of bounded monotone sequence. Limit inferior and limit superior of the sequences. Series of numbers, tests for convergence (without proof), test for absolute convergence, convergence of sequences of non-negative terms. (15L)

**Unit-3:** Real valued function, Continuous function, Uniform continuity of sequence of functions, Uniform convergence of power series, Radius of convergence. Reimann, Reimann-Stieltjes Integrals and their common properties. Integration by parts, Fundamental theorem on calculus, mean value theorem. (15L)

**Unit-4:** Vector and Matrix differentiation, Maxima, minima of functions of variables. Constrained maxima, minima, Lagrange's method, Taylor's theorem (without proof) and its applications. Implicit function theorem and their applications. Multiple Integrals, Change of variables, Improper integrals, Applications in multivariate distributions. Theorem on differentiation under integral sign (without proof), Leibnitz rule (statement only) and applications. (15L)

**Reference Books:**

1. Malik S. C. and Arora S. (1991). Mathematical Analysis- Wiley Eastern Limited 2<sup>nd</sup> Ed.
2. Goldberg R. R. (1964). Methods of Real Analysis-Blaisell Publishing company, New York, USA.
3. Bartle G.R. (1976). Element of Real Analysis-Wiley 2<sup>nd</sup> edition.
4. Royden (1988). Principles of Real Analysis-Macmillian.
5. Widder (1989). Advanced Calculus-Dover Publication.
6. Apostol T.M. (1985). Mathematical Analysis-Narosa Publishing House.
7. Walter Rudin (1976). Principles of Mathematical Analysis, McGraw-Hill Publication
8. Shanti Narayan and M.D. Raisinghanian (2003): Elements of Real Analysis, S. Chand Publication

|                  |   |                           |
|------------------|---|---------------------------|
| <b>Hard Core</b> | <b>HCT1.2: LINEAR ALGEBRA AND LINEAR MODELS</b> | <b>No. of credits: 04</b> |
|------------------|---|---------------------------|

**Unit-1:** Vector space, subspace, linear dependence and independence, basis, dimension of a vector space, example of vector spaces. Null space, Matrices, elementary operations, rank of a matrix and related results. Orthonormal basis and orthogonal projection of a vector, Orthogonal matrices, Gram-Schmidt orthogonalisation. Idempotent matrices, inverse of a matrix, their simple properties, partitioned matrices. Determinant of  $n \times n$  matrix. (15L)

**Unit-2:** G-inverse, Reduction of a matrix to echelon, diagonal, triangular forms, Hermitian matrices and its properties, Moore-Penrose inverse and its properties, Solution of a system of homogenous and non-homogenous linear equations and theorem related to existence of solution. (15L)

**Unit-3:** Characteristic roots of a matrix, algebraic and geometric multiplicities, characteristics vectors and their orthogonal property. Cayley-Hamilton theorem and applications. Spectral decomposition, singular value decomposition and Choleskey decomposition. Quadratic forms: Definition and classification, reduction, simultaneous reduction of two quadratic forms, maxima and minima of ratio of quadratic forms (15L)

**Unit-4:** General linear model: Definition, assumptions, concept of estimability, least squares estimation, BLUE, error space, estimation space, Guass-Markov theorem, variances and covariances of BLUEs. Distribution of quadratic forms in normal variables, related theorems (without proof). Tests of hypothesis in general linear models. One Way and two-way ANOVA. (15L)

**Reference Books:**

1. Rao A. R. and Bhimashankaram P. (1972): Linear Algebra, Tata McGraw Hill, New Delhi.
2. Hadely C. (1987): Linear Algebra, Narosa Publishing House.
3. Rao C. R. (1973): Linear Statistical Inference and Application, 2<sup>nd</sup> edition. John Wiley and Sons, Inc.
4. Searle S. R. (1982): Matrix Algebra useful for Statistics, John Wiley and Sons, Inc.
5. Graybill F. A. (1983): Matrices with application in Statistics- 2<sup>nd</sup> ed. Wadsworth
6. Kshirsagar A. M. (1983): Course in Linear Models-Marcel Dekker.

|                  |                                     |                           |
|------------------|-------------------------------------|---------------------------|
| <b>Hard Core</b> | <b>HCT 1.3: DISTRIBUTION THEORY</b> | <b>No. of Credits: 04</b> |
|------------------|-------------------------------------|---------------------------|

**Unit-1:** Brief review of basic distribution theory. Distribution function and its properties, Probability integral transform theorem. Symmetric distributions, properties of symmetric distributions, non-regular families, location and scale families and examples. Jordan decomposition theorem. Functions of random variables, their distributions in case of univariate random variables and its applications. (15L)

**Unit-2:** Expectation and moments, probability generating function, moment generating function, convolution and examples. Moment inequalities: Markov, Chebychev, Holder, Minkowski and Jensen's inequalities with their applications. Basic inequality of Liapunov's. (15L)

**Unit-3:** Order Statistics, their distributions and properties. Joint and marginal distributions of order statistics. Truncated distributions: Binomial, Poisson, Normal. Compound distributions and Mixture of distributions. Non-central chi-square, non-central t and non-central F distributions and their properties. (15L)

**Unit-4:** Bivariate discrete and continuous distributions, marginal distributions. Examples of joint distribution with given marginals, independence, conditional distributions and examples. Distribution function of bivariate random variable using Jacobian of transformation. Multinomial distribution, Bivariate Poisson, Bivariate exponential (Marshall and Olkin) and bivariate normal distributions and their properties. (15L)

**Reference Books:**

1. Rohtagi V.K. and Saleh A.K. Md. E (2001): Introduction to Probability Theory and Mathematical Statistics- John Wiley and Sons Inc.
2. Rao C. R. (1973): Linear Statistical Inference and Applications- John Wiley and Sons, Inc.
3. Johnson N. L. and Kotz S. (1996): Distribution in Statistics Vol-I, II and III- John Wiley and Sons, Inc.
4. Johnson N. L. and Kotz S.: Multivariate Distributions- John Wiley and Sons, Inc.
5. Casella G. and Berger R. L. (2002): Statistical Inference, Duxbury Advanced Series, 2<sup>nd</sup> Ed.

|                  |                                   |                           |
|------------------|-----------------------------------|---------------------------|
| <b>Hard Core</b> | <b>HCT 1.4: ESTIMATION THEORY</b> | <b>No. of credits: 04</b> |
|------------------|-----------------------------------|---------------------------|

**Unit-1:** Sufficiency principle, factorization theorem, minimal sufficiency, minimal sufficient partition, construction of minimal sufficient statistic, minimal sufficient statistic for exponential families, power series family and Pitman family. Completeness, bounded completeness, ancillary statistic, Basu's theorem and applications. (15L)

**Unit-2:** Problem of point estimation, unbiased estimators, minimum variance unbiased estimator, Rao-Blackwell and Lehmann-Scheffe theorems and their uses. Necessary and sufficient condition for MVUE. Fisher information and information matrix, Cramer-Rao inequality, Chapman-Robinson bound, Bhattacharya bound, their applications. (15L)

**Unit-3:** Methods of Estimation: Method of maximum likelihood estimation (MLE) and small sample properties of MLE, method of scoring and its application to estimation in multinomial distribution, Method of moments, Method of minimum chi-square. (15L)

**Unit-4:** Bayesian Estimation: The concept of prior distributions, various types of priors, posterior distribution, conjugate family and standard examples of such families. Bayes estimator under absolute and quadratic error loss functions. (15L)

**Reference Books:**

1. Rohtagi V.K. and Saleh A.K. Md. E (2001): Introduction to Probability Theory and Mathematical Statistics- John Wiley and Sons Inc.
2. Rao C.R. (1973): Linear Statistical Inference and Applications- John Wiley and Sons, Inc.
3. Casella G. and Berger R.L. (2002): Statistical Inference-Duxbury Advanced Series, 2<sup>nd</sup> ed.
4. Kale B.K. (2005): First Course on Parametric Inference- 2<sup>nd</sup> Eed. Narosa Publishing House.
5. Lehmann E.L. (1983): Theory of Point Estimation- John Wiley and Sons.
6. Ferguson T.S. (1967): Mathematical Statistics, Academic Press.
7. Rajgopalan M. and Dhanavanthan p. (2012): Statistical Inference-PHI Learning Pvt. Ltd.
8. Srivastava M. K., Khan A. H. and Srivastava N. (2014): Statistical Inference-Theory of Estimation- PHI Learning Pvt. Ltd.



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|------------------|---------------------------------------|----------------------------|
| <b>Soft Core</b> | <b>SCT 1.1: STATISTICAL COMPUTING</b> | <b>No. of credits : 04</b> |
|------------------|---------------------------------------|----------------------------|

**Unit-1:** Numerical Methods: Newton Raphson method, Bisection method, Regula falsi method, gradient search method. Numerical integration using Trapezoidal rule and Simpson's rule for single and double integrals. Programming exercise on these methods. Numerical algorithms such as direct search, grid search, interpolation search, gradient search, Bisection and Newton-Raphson methods, Mullers method, Aitkens extrapolation, Simple applications of the above methods. (15L)

**Unit-2:** Concept of Random Number Generator, Congruential Method of generating uniform variates, Concept of Simulation, Generation of random sample using inverse transform technique, acceptance-rejection technique, composition, convolution methods. Generation random sample from discrete distributions covering Binomial, Poisson, Geometric, Negative binomial , Multinomial and Bivariate Poisson. Generation random sample from continuous distributions covering Exponential, Normal, Gamma, Chi-square, Bivariate Normal, Bivariate Exponential distributions and Mixture of distributions. (15L)

**Unit-3** Methods to compute integrals- quadrature formula, double integration, Gaussian integration, Monte Carlo Methods: Monte Carlo integration and its application to compute expected values and probabilities, Theory of Importance Sampling with applications to reduce Monte Carlo error and rare-event simulation, Verification of WLLN, CLT and other approximations through simulation. (15L)

**Unit-4:** Bias Reduction Methods, Jack-Knife estimator, its properties and limitations. Bootstrap method and its simple properties. Introduction to EM algorithm, Markov Chain Monte Carlo method. (15L)

### Reference Books

1. Ryan B. and Joiner B. L. (2001). MINITAB Handbook, 4th Ed. Duxbury.
2. Thisted R. A. (1998). Elements of Statistical Computing, Chapman and Hall.
3. Kennedy William J. Jr., James E. Gentle (1980), Statistical Computing, Marcel Dekkar
4. Morgan B. J.T. (1984), Elements of Simulation, Chapman and Hall.
5. Purohit, Gore, and Deshmukh (2008), Statistics Using R, Alpha Science International

|           |                     |                    |
|-----------|---------------------|--------------------|
| Soft Core | SCT 1.2: DEMOGRAPHY | No. of credits: 04 |
|-----------|---------------------|--------------------|

**Unit-1:** Demography and its interdisciplinary nature, sources of demographic data, Coverage and Content errors. The use of balancing equation, Chandrasekaran and Deming formula to check completeness of registration data. Use of Whipple's Myers's and UN Indices. Mixture of distributions. Proofs of related results. (15L)

**Unit-2:** Measures of Mortality: Various measures of mortality, infant mortality rate, cause specific death rates and standardized death rates. Measures of Fertility: Period and cohort fertility measures, use of birth order statistics, child-women ratio, Brass P/F ratio to estimate current level of fertility, Measures of reproduction and replacement. Sheps and Perrin stochastic human reproductive process. (15L)

**Unit-3:** Life Tables: Types of life tables, inter-relationships between life table functions, construction of life tables using Reed- Meerel and Greville's Method. Probability distribution of life table functions and their optimum properties.

Population estimation and Projection: Mathematical, Statistical and Demographic Methods, Component method. (15L)

**Unit-4:** Stable and Quasi-stable population: Derivation of Lotka's stable population model and properties, Intrinsic growth rate and derivation, age structure and birth rate of a stable population, mean length of generation, momentum of population growth, Quasi-stable population under changing fertility and mortality situations. (15L)

**Reference Books:**

1. Shryoch, Henry S, Jacob S, Siegel and Associates (1964): Methods and materials of demography (condensed edition) Academic press, London.
2. Barclay, George W. (1968): Techniques of population analysis, John Wiley and Sons, New York.
3. Keyfitz N. (1968): Introduction to Mathematics of Population. Addison-Wesley Publishing Co, Reading, Massachusetts.
4. R. Ramkumar (1986): Technical Demography, Wiley Eastern, New Delhi.
5. Sudhendu Biswas (1988): Stochastic processes in Demography and Applications, Wiley Eastern, New Delhi

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|------------------|--|---------------------------|
| <b>Hard Core</b> | <b>HCP 1.1: STATISTICS PRACTICAL-1</b><br>(Based on HCT 1.2 and HCT 1.4) | <b>No. of credits: 02</b> |
|------------------|--|---------------------------|

**Use of Statistical Software Packages:**

- MINITAB Software
- R Software
- SPSS
- SYSTAT
- SAS

This paper includes practical problems from papers HCT 1.2 and HCT 1.4. There will be at least ten practicals. Each practical should consist of problems to be solved using each of the following software EXCEL/ R/ MINITAB/ MATLAB/ SYSTAT wherever applicable.

|                  |   |                            |
|------------------|---|----------------------------|
| <b>Hard Core</b> | <b>HCP 1.2 STATISTICS PRACTICAL-2</b><br>(Based on HCT 1.3 and SCT 1.1/1.2) | <b>No. of credits : 02</b> |
|------------------|---|----------------------------|

**Use of Statistical Software Packages:**

- MINITAB Software
- R Software
- SPSS
- SYSTAT
- SAS

This paper includes practical problems from papers HCT 1.3 and SCT1.1/ SCT1.2. There will be at least ten practicals. Each practical should consist of problems to be solved using each of the following software EXCEL/ R/ MINITAB/ MATLAB/ SYSTAT wherever applicable.

## M.Sc. (STATISTICS) Semester-II

|                  |                                    |                           |
|------------------|------------------------------------|---------------------------|
| <b>Hard Core</b> | <b>HCT 2.1: PROBABILITY THEORY</b> | <b>No. of credits: 04</b> |
|------------------|------------------------------------|---------------------------|

**Unit-1:** Classes of sets, Sequence of sets, limsup and liminf and limit of sequence of sets, field,  $\sigma$ - field,  $\sigma$ - field generated by a class, Borel  $\sigma$ -field. Probability measure, Probability space, properties of probability measure-continuity, mixture of probability measures. Lebesgue and Lebesgue-Stieltjes measures on  $\mathbb{R}$ . Independence of events. (15L)

**Unit-2:** Measurable function, random variable, distribution of a random variable, simple random variable, elementary random variable, liminf, limsup and limit of sequence of random variables. Method of obtaining a random variable as a limit of sequence of simple random variables. Integration of a measurable function with respect to a measure. Expectation of a random variable, independence. Characteristic function, simple properties, Inversion theorem and uniqueness property (Statement only). (15L)

**Unit-3:** Monotone convergence theorem, Fatou's Lemma, Dominated Convergence theorem, Borel-Cantelli Lemma and their applications. Convergence of sequence of random variables, Convergence of distribution, Continuity theorem (Statement only), Almost sure convergence, a characterizing property, convergence in probability, uniqueness of limit. Yule-Slutsky results. convergence in  $r^{\text{th}}$  mean, interrelationships. (15L)

**Unit-4:** Weak and Strong laws of large numbers, Kolmogorov's three series theorem for almost sure convergence (statement only), Liapoune's Lindeberg-Feller Theorems on CLT (statement only). Application of the above results. (15L)

### Reference Books:

1. Bhat B. R. (1999): Modern Probability Theory (3<sup>rd</sup> Ed.), New Age International (P) Ltd.
2. Billingsley P. (1986): Probability and Measure, John Wiley and Sons.
3. Chandra, T. and Gangopadhyay, S. (2017): Fundamentals of Probability Theory, Narosa Publishing House.
4. Gut, A. (2005): Probability: A Graduate Course, Springer.
5. Alan Karr (1993): Probability Theory, Springer Verlag.
6. Kingman, J F C and Taylor, S.J. (1966): Introduction to Measure and Probability, Cambridge University Press.
7. Dudley R.M. (1989): Real Analysis and Probability, Wadsworth and Brooks/ Cole.
8. Ash Robert (1972): Real Analysis and Probability, Academic Press.

|                |                             |                            |
|----------------|-----------------------------|----------------------------|
| <b>HCT 2.2</b> | <b>STOCHASTIC PROCESSES</b> | <b>No. of credits : 04</b> |
|----------------|-----------------------------|----------------------------|

**Unit-1:** Definition of Stochastic process, Finite dimensional distributions. Examples of various stochastic processes. Markov chain and its Examples. Chapman-Kolmogorov equation, calculation of n-step transition probabilities. Classification of states of Markov chain, irreducible Markov chain, period of the state, First entrance theorem, First passage time distribution. Long run distribution of Markov chain, relation of mean recurrence time and stationary distribution. (15L)

**Unit-2:** Random walk and gambler's ruin problem. Discrete state space continuous time Markov chain. Poisson process and related results. Birth and death processes and associated cases. M/M/1, M/M/S queuing models and related properties. (15L)

**Unit-3:** Galton-Watson branching process. Probability of ultimate extinction, distribution of population size and associated results. Simulation of Markov chain, Poisson process, branching process (Algorithms) (15L)

**Unit-4:** Brownian motion, Hitting Time, Maximum, Reflection Principle Stopping Time and Strong Markov Property, Reflection Principle. (15L)

**Reference Books:**

1. Medhi. J. (1982). Stochastic Process- Wiley Eastern.
2. Parzen E. (1962). Stochastic Process-Holden-Pay.
3. Karlin & Taylor (1975). A First Course in Stochastic Processes-Vol-I Academic Press.
4. Cinlar E. (1975). Introduction to Stochastic Process. Prentice Hall.
5. Srinivas and Mehta (1976). Stochastic Processes-Tata McGraw-Hill.
6. Adke and Manjunath (1984). An introduction to finite Markov Processes- Wiley Eastern.
7. Bhat B. R. (2000). Stochastic Model: Analysis and Application, New Age International.
8. Sheldon Ross: Introduction to Probability Models, 11<sup>th</sup> Edition, Academic Press Publication

|                  |   |                           |
|------------------|---|---------------------------|
| <b>Hard Core</b> | <b>HCT 2.3: THEORY OF TESTING OF HYPOTHESES</b> | <b>No. of credits: 04</b> |
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**Unit-1:** Problem of testing of hypothesis: Simple and Composite hypotheses. Randomized and non-randomized tests. Power function, power function of a test, Most powerful test, Neyman-Pearson Lemma and its applications. Determination of minimum sample size to achieve the desired strengths. (15L)

**Unit-2:** Monotone likelihood ratio (MLR) property, UMP tests and their existence for one-sided alternatives. UMP tests for two sided alternatives: their existence and non-existence. Examples. Generalized Neyman Pearson Lemma, Unbiased test, UMPU tests and their existence in the case of exponential families (Statements of the theorems only), Similar tests, Test with Neyman structure. Likelihood ratio test, application to standard distributions. (15L)

**Unit-3:** Interval estimation, confidence level, construction of confidence intervals using pivots, shortest length confidence interval, UMA confidence interval and its relation to UMP test, UMAU confidence interval and its relation with UMPU test. (15L)

**Unit-4:** Definition of U-statistics, U-statistics theorem for one sample and two samples (statements only). Goodness of fit test based on chi-square distribution, application to contingency tables. Non-parametric Tests: One sample tests: Sign test, Wilcoxon signed-rank test, Run test. Two sample tests: Wald-Wolfowitz runs test, Mann-Whitney U test, Kolmogorov-Smirnov test, Tests for two-sample scale problem: Mood, Klotz, Ansari-Bradley, Siegel-Tukey and Sukhatme tests. Spearman's rank correlation test, Kendall's rank correlation test, Kruskal-Wallis Test. (15L)

**Reference Books:**

1. Kale B. K. (1999). A first Course on Parametric Inference, Narosa
2. Rohatgi V.K. (1988), Introduction to Probability and Mathematical Statistics, Wiley Eastern Ltd. New Delhi. Student Edition.
3. Dudewicz E. J. and Mishra S. N. (1988), Modern Mathematical Statistics, Wiley Series
4. Lehman E. L. (1987). Theory of Testing of Hypotheses. Student Edition.
5. Srivastava M. and Srivastava N. (2009): Statistical Inference-Testing of Hypotheses, PHI Learning Pvt. Ltd.
6. Shanthakumaran A. (2001): Fundamentals of Testing of Hypotheses, Atlantic Publishers and Distributors.
7. Gibbons J. D. and Chakraborti S. (2003): Nonparametric Statistical Inference, 2nd Edition, Marcel Dekker, Inc.
8. Puri M. L. and Sen P. K. (1971): Nonparametric Methods in Multivariate Analysis, John Wiley and Sons.

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| <b>Soft Core</b> | <b>SCT 2.1: SAMPLING THEORY</b> | <b>No. of credits : 04</b> |
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**Unit-1:** Review of concept of population and sample, Need for sampling, Census and sample surveys, basic concepts in sampling and designing of large-scale survey design, sampling scheme and sampling strategy. Simple random sampling with and without replacement (SRSWR and SRSWOR). (15L)

**Unit-2:** Stratified Sampling: Stratification, Allocation and estimation problems, Construction of Strata, deep stratification, method of collapsed strata. Systematic sampling: The sample mean and its variance, comparison of systematic sampling with random sampling, comparison of systematic sampling with stratified sampling, comparison of systematic sampling with simple and stratified random sampling for certain specified population. (15L)

**Unit-3:** PPSWR Methods: Cumulative total method, Lahiri's method, related estimation problems, PPSWOR method and related estimation of a finite population mean (Horvitz-Thompson and Des Raj estimator for general sample size and Murthy's estimator for sample of size 2, Midzuno sampling, Rao-Hartley-Cochran sampling strategy, Poisson and modified Poisson sampling strategy. (15L)

**Unit-4:** Use of supplementary information for estimation: Ratio and Regression estimators and their properties. Unbiased and almost Unbiased ratio type estimators, Double sampling, Cluster sampling, Two-stage sampling with equal number of second stage units. Non-sampling errors, Response and non-response errors, Hansen-Hurwitz and Deming's techniques. (15L)

**Reference Books:**

1. Sukhatme P.V., Sukhatme P.V., Sukhatme S. and Ashok C. (1997): Sampling Theory of Surveys and Applications - Piyush Publications.
2. Des Raj and Chandhok P. (1998): Sample Survey Theory. Narosa publishing House.
3. William G. Cochran (1977): Sampling Techniques, 3<sup>rd</sup> edition- John Wiley & Sons.
4. Parimal Mukhopadhyay (1988): Theory and methods of Survey sampling, Prentice Hall of India Pvt. Ltd.
5. Murthy M.N. (1977): Sampling Theory and Methods, Statistical publishing Society, Calcutta.

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| <b>Soft Core</b> | <b>SCT 2.2: ACTUARIAL STATISTICS</b> | <b>No. of credits: 04</b> |
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**Unit-1:** Future life time random variable, its distribution function and density function, concept of force of mortality, curtate future life time random variable its probability mass function, deferred probabilities, all these functions in terms of international actuarial notation. Analytical laws of mortality such as Gompertz' law and Makeham's law, single decrement life table, select and ultimate life table. (15L)

**Unit-2:** Concept of compound interest rate, discount factor, present value of the money, nominal rate of interest, force of interest. Assurance contracts with level and varying benefits, such as whole life insurance, term insurance endowment insurance. Means and variances of the present value random variables of the payments under these contracts under the assumption of constant force of interest, when the benefit payments are made at the end of year (discrete set up) or when it is paid at the epoch of death(continuous set up). Actuarial present value of the benefit. Net single premiums. (15L)

**Unit-3:** Annuity contracts, annuity certain, discrete annuity, m-thly annuity, continuous annuity, deferred annuity, present values and accumulated values of these annuities. Continuous and discrete life annuity, such as whole life annuity, temporary life annuity, n-year certain and life annuity, life annuities with mthly payments. Present value random variables for these annuity payments, means and variances. Actuarial present value of the annuity. (15L)

**Unit-4:** Loss at issue random variable, various principles to decide net premiums for insurance products and annuity schemes defined in unit II and III, fully continuous premiums and fully discrete premiums, True m-thly payment premiums. Extended equivalence principle to decide gross premiums. Concept of reserve, prospective and retrospective approach. Fully continuous reserve. Fully discrete reserve. (15L)

**Reference Books:**

1. Bowers, JR. N.L., Gerber, H.U., Hickman, J.C., Jones, D.A. and Nesbitt, C.J. (1997): Actuarial Mathematics, 2nd Edition, The Society of Actuaries.
2. Deshmukh S.R. (2009): Actuarial Statistics: An Introduction Using R, Universities Press.
3. Harriett, E.J. and Dani, L. L.(1999): Principles of Insurance: Life, Health, and Annuities, 2nd Edition, Life Office Management Association.
4. Neill, Alistair (1977): Life Contingencies, The Institute of Actuaries.
5. Palande, P. S., Shah, R. S. and Lunawat, M. L. (2003): Insurance in India-Changing Policies and Emerging Opportunities, Response Books.



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| <b>Open Elective</b> | <b>OET 2.1: STATISTICAL METHODS</b> | <b>No. of credits: 04</b> |
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**Unit-1:** Descriptive Statistics: Measures of central tendency, arithmetic mean, geometric mean, harmonic mean, median and mode for grouped and ungrouped data with examples. Measures of dispersion, range, quartile deviation, variance, standard deviation, coefficient of variation, skewness and kurtosis. Examples and problems. (15L)

**Unit-2:** Correlation and regression: Scatter diagram, Karl Pearson's coefficient of correlation, rank correlation, regression, lines of regression, regression coefficients, fitting of regression lines. Examples and problems. (15L)

**Unit-3:** Probability and Probability distributions: Random experiment, Trial, Sample space, Sample point and different types of events. Classical definition of Probability, addition and multiplication rules. Random variable (discrete and continuous) and its probability distribution. Probability mass function and Probability density function. Bernoulli, Binomial, Poisson, Uniform, Exponential and Normal distributions, their means and variances. (15L)

**Unit-4:** Testing of hypothesis: Notion of hypothesis, null and alternative hypothesis, simple and composite hypothesis, test statistic, critical region, idea of one and two tailed test, type-I and type-II errors, level of significance. Hypothesis testing for mean and proportion. Hypothesis testing for difference of two means and two proportions. Chi-square test for independence of attributes. Nonparametric run test, sign test and signed-rank test. (15L)

**Reference Books:**

1. Gupta S.C. and Kapoor V. K.: Fundamentals of Mathematical Statistics. Sultan Chand and Sons Publications, New Delhi.
2. Kapoor J. N and Saxana H.C.; Mathematical Statistics, Sultan Chand & sons Publications, New Delhi
3. Kulkarni M.B., Gore A.P. and Ghatpande S.B.: Statistical Tests, Satyajeet Prakashan, Pune.
4. David D. Hanagal (2009): Introduction to Applied Statistics: A Non-Calculus Based Approach

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| <b>Open Elective</b> | <b>OET 2.2: MATHEMATICAL STATISTICS</b> | <b>No. of credits: 04</b> |
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**Unit-1:** Probability theory: Sample space, Axioms of probability theory. Conditional probability, independent events. Baye's theorem, Examples.

Discrete random variables : Definition of a random variable, Probability mass functions and probability distribution functions, Bernoulli trials and related distributions, Poisson distribution, Discrete Uniform distribution, geometric distribution, Expectation and moments and their evaluation, Examples. (15L)

**Unit-2:** Continuous random variables: Definition of a continuous random variable, Probability density function. Expectation and moments and their evaluation, uniform, exponential and gamma distributions, normal distribution. Distribution of functions of a continuous random variable, Examples. (15L)

**Unit-3:** Bivariate random variables: Discrete bivariate distributions, continuous bivariate distributions. Covariance and correlation. Conditional distribution and conditional mean, Bivariate normal distribution, Examples. (15L)

**Unit-4:** Generating functions: Probability generating function; moment generating function; and characteristic function; their properties, Examples.

Moment inequalities: Markov, Chebychev, Holder, Minkowski and Jensen inequalities with their applications. (15L)

**Reference Books:**

1. Gupta S.C. and Kapoor V.K.: Fundamentals of Mathematical Statistics. Sultan Chand & Sons Publications, New Delhi.
2. Kapoor J.N and Saxana H.C : Mathematical Statistics, Sultan Chand & Sons Publications, New Delhi.
3. Bhat B. R. (1999): Modern Probability Theory (3<sup>rd</sup> ed.), New Age International (P) Ltd.

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| <b>Hard Core</b> | <b>HCP 2.1: STATISTICS PRACTICAL-3</b> | <b>No. of Credits: 02</b> |
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**Use of Statistical Software Packages:**

- MINITAB Software
- MATLAB Software
- R Software
- SPSS
- SYSTAT

This paper includes practical problems from papers HCT 2.1, HCT 2.2 and SCT2.1/ SCT2.2. There will be at least ten practicals. Each practical should consist of problems to be solved using each of the following software EXCEL/ R/ MINITAB/ MATLAB/ SYSTAT wherever applicable.

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| <b>Open Elective</b> | <b>OEP 2.1: STATISTICS PRACTICAL-4</b> (Based on OET 2.1)<br><b>OR</b><br><b>OEP 2.2: STATISTICS PRACTICAL-4</b> (Based on OET 2.2) | <b>No. of credits: 02</b> |
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**Use of Statistical Software Packages:**

- MS-EXCEL Software

This paper includes practical problems from papers OET2.1/OET2.2. There will be at least eight practicals. Each practical should consist of problems to be solved using MS-EXCEL or Scientific Calculator.